WIRELESS ENGINEER

THE JOURNAL OF RADIO RESEARCH & PROGRESS

APRIL 1956

VOL. 33 No. 4 · THREE SHILLINGS AND SIXPENCE

MODEL 7 — 50 RANGE

Universal AvoMeter

Fifty ranges...

in one instrument

THE wide scope of this multi-range AC/DC measuring instrument, coupled with its unfailing reliability, simplicity of use and high degree of accuracy, renders it invaluable wherever electrical equipment has to be maintained in constant, trouble-free operation.

CURRENT	VOLTAGE
1mA D.C. only	50mV D.C. only
2 ,, ,, ,,	100 ,, ,, ,,
5 ,, A.C. & D.C.	500 ,, ,, ,,
10 ,, ,, ,,	1V ,, ,,
50 ,, ,, ,,	5 ,, A.C. & D.C.
100 ,, ,, ,,	10 ,, ,, ,,
500 ,, ,, ,,	50 ,, ,, ,,
1 A ,, ,,	100 ,, ,, ,,
5 ,, ,, ,,	200 ,, ,, ,,
10 ,, ,, ,,	400 ,, ,, ,,
	500 ,, ,, ,,
	1,000 ,, ,, ,,

RESISTANCE

10,000 ohms, 100,000 ,, } using internal 1½ volt cell

1 Megohm — using internal 9 volt battery

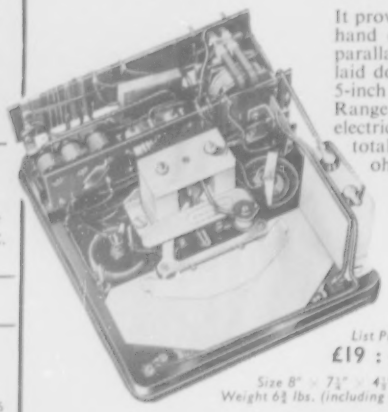
10 Megohms, 40 ,, } using external source of A.C. or D.C. voltage

CAPACITY 0·01—20 mFds

POWER AND DECIBELS

Impedance	Power	Decibels 0 50W
500 ohms	200 mW	—25 to + 6
5,000 ohms	2W	—15 to + 16
50,000 ohms	200 mW	—25 to + 6

Various accessories are available for extending the wide range of measurements.



It provides 50 ranges of readings on a 5-inch hand calibrated scale fitted with an anti-parallax mirror. Accuracy is within the limits laid down in Section 6 of B.S.S. 89/1954 for 5-inch scale industrial portable instruments. Range selection is effected by means of two electrically interlocked rotary switches. The total resistance of the meter is 500,000 ohms.

The instrument is self-contained, compact and portable, simple to operate, and is protected by an automatic cut-out against damage through inadvertent overload.

Power and Power Factor can be measured in A.C. circuits by means of an external accessory, the Universal AvoMeter Power Factor & Wattage Unit.

List Price

£19 : 10s.

Size 8" × 7¼" × 4½"
Weight 6¾ lbs. (including leads)

...you can depend on AVO

Illustrated Brochure available on request.

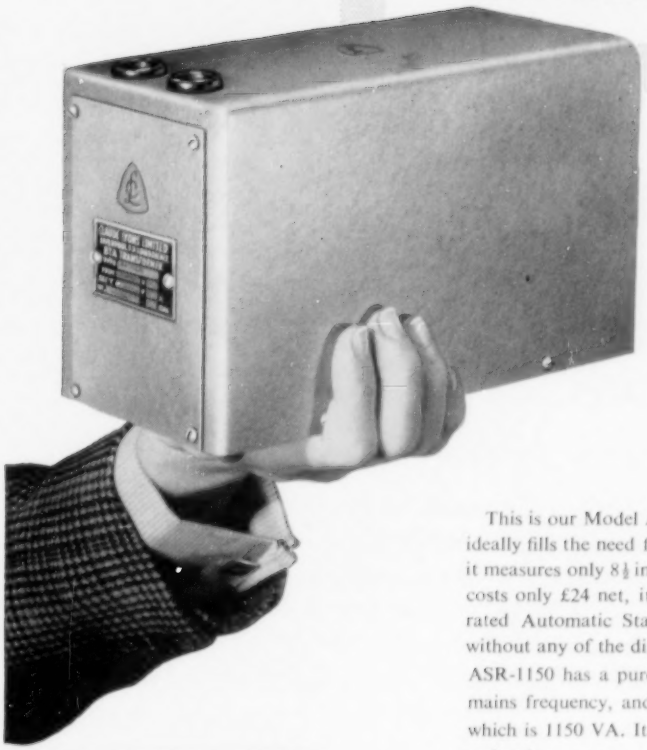
Sole Proprietors and Manufacturers:—

THE AUTOMATIC COIL WINDER & ELECTRICAL EQUIPMENT CO., LTD.

AVOCET HOUSE · 92-96 VAUXHALL BRIDGE ROAD · LONDON · S.W.1. *Telephone: VICtoria 3404 (9 lines)*

A7/9

A NEW-PRINCIPLE A-C AUTOMATIC VOLTAGE STABILISER



This is our Model ASR-1150 Automatic Voltage Step Regulator. It ideally fills the need for a cheap, small and light Stabiliser. Although it measures only 8½ in. × 4½ in. × 5 in., weighs as little as 11 lb., and costs only £24 net, it has a performance fully equal to any similarly rated Automatic Stabiliser of the resonated, saturated core type, without any of the disadvantages.

ASR-1150 has a pure output waveform, is unaffected by changes in mains frequency, and works equally well from no-load to full load, which is 1150 VA. It has a stabilised output at 230V unless otherwise ordered.

Many other Automatic Voltage Stabilisers are now manufactured by us, and all are available for immediate delivery. In some cases the constancy of output is as high as 0.15%. Models are available from 200 VA to 30 kVA, single phase. 3-Phase Stabilisers are also available. Prices are *extremely* competitive.

We can supply from stock all types of American tubes, condensers, valves, potentiometers, etc.

MEMO: If you are interested in infinitely-variable Transformers, do not forget the almost indispensable "VARIAC" (Reg'd. Trademark). Models are available from 170VA to 21kVA. Our Catalogue V-549 (3rd Edition) tells the whole story and will gladly be mailed free and post free, on request.

The *NEW "ASR-1150"* costs only *£24* net

Complete information is obtainable from:

CLAUDE LYONS LTD., STABILISER DIVISION,

HEAD OFFICES & WORKS: 76 OLDHALL STREET, LIVERPOOL 3, LANCS.
SOUTHERN FACTORY: VALLEY WORKS, WARE ROAD, HODDESDON, HERTS.
(A10 main London/Cambridge Road, at junction of A602)

PHYSICAL SOCIETY 1956

Cinema-Television Limited announce the first demonstrations of four new instruments which will be given at the above exhibition.

HIGH SPEED OSCILLOSCOPE

Employing a new 5" diameter 'Cintel' tube this oscilloscope is designed for repetitive and single shot pulse examination and has a frequency response extending to 500Mc/s. Sensitivity 50V/inch. Maximum writing speed 50 inches/μsec.

DECIMICROSECOND CHRONOMETER

A high speed chronometer for measuring time in the range 0.125μsec to 1μsec in steps of 0.125μsec. Indication of the measurement is given on seven panel mounted meters in decimal notation.

ELECTRONIC PHOTOGRAPHIC PRINTER

Designed to provide uniform contrast contact prints from variable density negatives without loss of information.

SIGNAL TRACER

A multi-purpose instrument designed on the well known 'Grid Dip' principle.

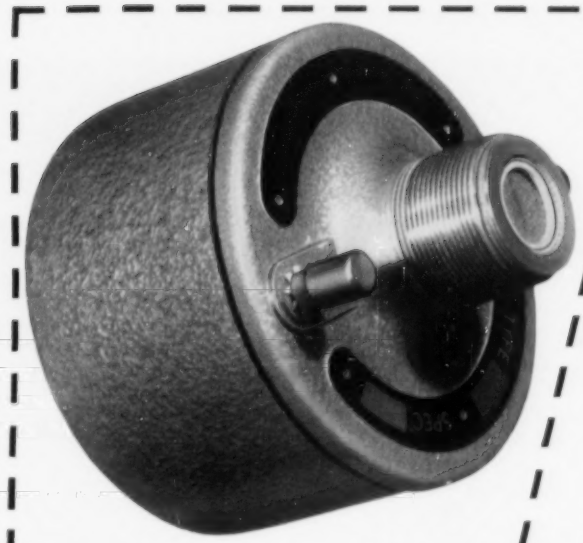
The following will also be demonstrated:

Miniature Multi-channel Recording Equipment.
Flying-Spot Microscope with Counter.
Automatic Frequency Monitor 20Mc/s,
and a range of specialized Cathode Ray Tubes.

CINEMA TELEVISION LTD

A COMPANY WITHIN THE RANK ORGANISATION LIMITED

WORSLEY BRIDGE ROAD · LONDON · S.E.26
HITHER GREEN 4600



DC 10
PRESSURE UNIT

SPECIFICATION

Power Handling Capacity	10 watts peak
Voice Call Impedance	15 ohms
Frequency Response	120–5000 c.p.s.
Flux Density	12,000 gauss
Pole Piece	$\frac{1}{2}$" diameter

Price £6 10s. 0d.

DIMENSIONS

	DC 10 (without line trs.)	DC 12 (with line trs.)
Diameter	$4\frac{1}{8}$"	$4\frac{1}{8}$"
Length	$4\frac{1}{4}$"	$6\frac{3}{8}$"
Weight	$4\frac{3}{4}$lb.	$5\frac{1}{2}$lb.

Specially designed and developed to meet the need of the P.A. Engineer requiring a compact, efficient unit combining good tone with average handling capacity, at a price that will make the "small" installation a profitable proposition.

High Sensitivity Heavy cross-sectioned cup with latest anisotropic alloy CP magnet.

Phase Equalizing Throat One-piece zinc based alloy die-casting.

Self-Centring Diaphragm Assembly Can be changed in the field without special tools or soldering iron in $1\frac{1}{2}$ mins.

Spring Loaded Terminals Ensure quick and positive line termination.

Weatherproof Totally enclosed, ensuring protection when in exposed position, watertight gland cable entry.

DC 12 PRESSURE UNIT (WITH TRANSFORMER)
As DC 10 but fitted with totally enclosed 100v line transformer tapped 2·5, 5, 8 and 12 watts.
PRICE £7 10s. 0d.

Rola Celestion Ltd.
FERRY WORKS THAMES DITTON, SURREY
Telephone: EMBerbrook 3402

G.E.C.

ALUMINISED C.R. TUBES

for tube replacements

Price £14 . 15 . 0
P. Tax £6 . 18 . 1

The G.E.C. 14 in. Cathode Ray Tube type 7203A has the following attractive features:—

1) Improved tube life and reliability.

2) A narrow beam angle *triode* gun giving:—

(a) Uniform focus over the entire screen area.

(b) Small spot size.

(c) Freedom from astigmatism.

3) A high reflectance aluminium backing to the fluorescent screen giving:—

(a) 70% increase in picture brightness.

(b) High contrast daylight viewing.

(c) Complete elimination of ion burn from both positive and negative ions.

THE GENERAL ELECTRIC CO. LTD., MAGNET HOUSE, KINGSWAY, LONDON, W.C.2

for radio ceramics

STEATITE & PORCELAIN PRODUCTS LTD.

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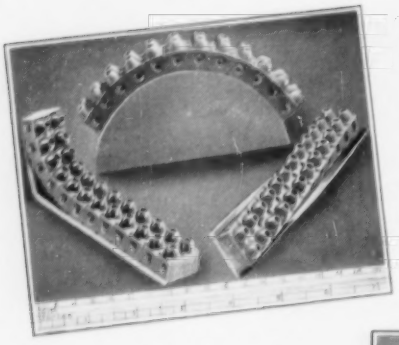
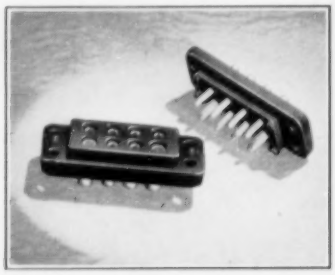
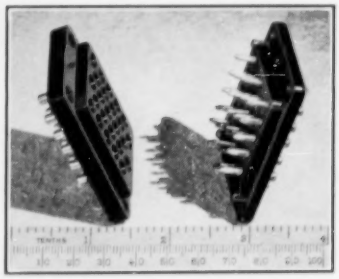


'UNITORS'

inter-unit connectors for 4 to 25 contacts





"BELLING-LEE"

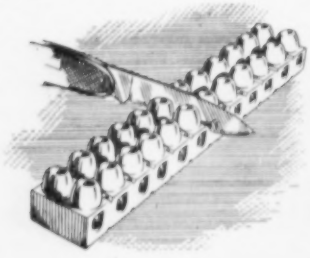
FLEXIBLE TERMINAL BLOCK

U.K. Patent 680406

This unique component was designed to provide great flexibility in both planes, thus enabling it to be secured to curves or irregular surfaces. It is moulded in P.V.C., which securely grips the terminal screws so that they cannot fall out **even if totally unscrewed from their inserts and the block mounted upside down.**

The block is mechanically shockproof and can easily be sub-divided with an ordinary knife into smaller groups of terminals. Fixing holes provided for each pair of terminals.

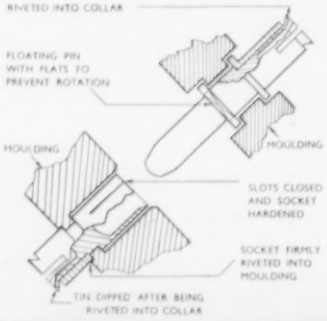
L.744 is a 12-way strip, rated at 5 amp., but may be used at the designer's discretion up to 10 amp., or up to 2 kV peak working voltage between terminals or between terminals or chassis. Overall size, 5 in. × .750 in. × .625 in. high. Hole centres spaced .425 in. L.1325 is a new larger version, rated at 15 amp., and conforms with safety requirements B.S.A.15, at 500 v. (r.m.s.).



This popular range owes its origin to a very successful Government development contract. They conform to R.C.S. 321 and have Joint Service and A.R.B. approval. Pins carry 3 amp., but each block has two larger pins carrying 10 amp. each. All pins are fully floating.

Each coupling consists of a block of plugs and one of sockets, arranged so as to be non-reversible. Bodies are moisture and tracking-resistant, being moulded from a nylon-filled phenolic material. Pins are of high grade brass silver-plated. Sockets are of differentially hardened beryllium copper; solder spills of both plugs and sockets are slit to facilitate soldering. All plugs and sockets are numbered on both sides of the moulded body.

Die-cast light alloy covers provide a flex lead connector. Newer types of covers and retainers are under development and these will be freely interchangeable with all existing types, which will be continued as standard until the new versions are available.


RIVETED INTO COLLAR
FLOATING PIN WITH FLATS TO PREVENT ROTATION
MOULDING
MOULDING
SLOTS CLOSED AND SOCKET HARDENED
SOCKET FIRMLY RIVETED INTO MOULDING
TIN DIPPED AFTER BEING RIVETED INTO COLLAR

U.K. Pat. 649739. French Pat. 990017. Swiss Pat. 277799

LIST NO.	PINS	BODY SIZE
L.653/P&S	4	1.7/16"×13/32"
L.654/P&S	8	1.31/64"×37/64"
L.655/P&S	12	1¼"×25/32"
L.656/P&S	18	2"×27/32"
L.657/P&S	25	1¼"×1.3/16"

Working volts:
500V. Peak, pin to pin

BELLING & LEE LTD
GREAT CAMBRIDGE RD., ENFIELD, MIDDX., ENGLAND

Telephone: Enfield 3322 **Telegrams: Radiobel, Enfield**

Our research and development departments are continually engaged on research work in connection with powdered magnetic materials to bring to you the finest quality cores available

Write for list No. GRC. 5303/6.

SALFORD ELECTRICAL INSTRUMENTS LTD., SALFORD 3, LANCS.

London Office: MAGNET HOUSE, KINGSWAY, W.C.2 TEMple Bar 4669

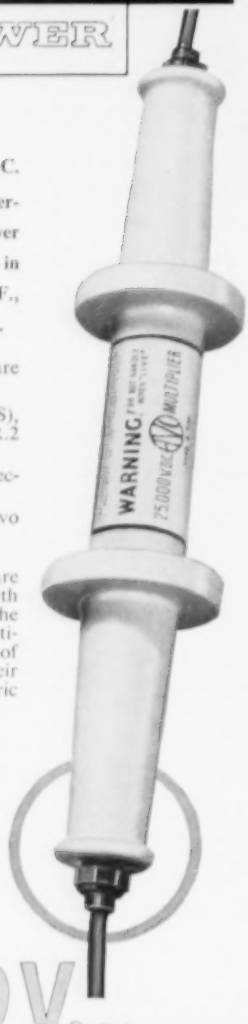
A Subsidiary of THE GENERAL ELECTRIC CO. LTD. OF ENGLAND

For measuring

HIGH VOLTAGE

at

LOW POWER



The "Avo" 25,000-volt D.C. Multipliers are intended to operate with high voltage, low power systems similar to those found in television receivers, e.g: R.F., E.H.T. and line fly-back units.

Three types of Multiplier are available:-

Type 8 for models 8, 8(S), 8(S)X, 8X, HR.1 and HR.2 AvoMeters.

Type ETM for the Avo Electronic Testmeter.

Type CT38 for the Avo Electronic Multimeter.

In use, these accessories are connected into the circuit with the indicating instrument at the low potential end of the Multiplier. They are constructed of materials chosen for their rigidity and high dielectric strength.

Particular attention has been devoted to the problem of corona discharge and the voltage gradient across the resistive unit.

AVO

25,000V.

D.C. Multipliers

List Price: £7 : 10s.

Write for fully descriptive leaflet:

THE AUTOMATIC COIL WINDER & ELECTRICAL EQUIPMENT CO. LTD.

AVOCET HOUSE · 92-96 VAUXHALL BRIDGE ROAD LONDON · S.W.1. 'Phone: VICtoria 3404 (9 lines)

DCM.1

TRUVOX

Tape Recorder

Model R1

(Release Mid-July)

It has taken a long time to produce a Recorder that is really worthy of the name TRUVOX and all that the name implies, but it has come—an instrument that does full justice to the world-famous Truvox Tape Recording products. A high-fidelity instrument in every sense of the word, giving not only endless hours of enjoyment recording and replaying one's own programmes but also providing perfect reproduction of the new, outstanding and ever-growing number of pre-recorded tapes of the world's finest music, with all those finer gradations of tone that are there *for those who can hear the difference.*

Contemporary cabinet styling is in keeping with the high standard set by the equipment it contains—a rich burgundy leatherette set off by gold-hammered metal work.

Price complete with crystal microphone and 1,200 ft. reel of standard tape. **66 GNS**

Additional optional equipment

Truvox Senior Radio Jack
Truvox Standard Radio Jack
Truvox Foot Control
Truvox Headphones for transcription of recorded material.
Truvox telephone attachment for recording two-way telephone conversations.

A TRUVOX PRODUCT

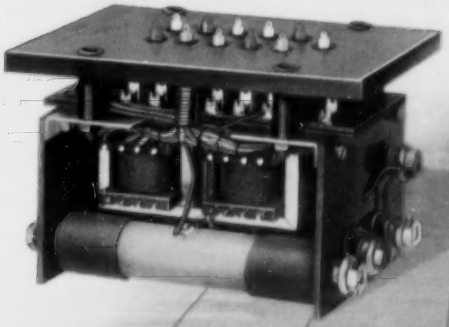
Send for full details

TRUVOX LIMITED

Sales Office: 15 Lyon Road, Harrow, Middlesex
Harrow 9282

Tech. & Service Dept.: 328 Station Road, Harrow, Middlesex. Harrow 4455

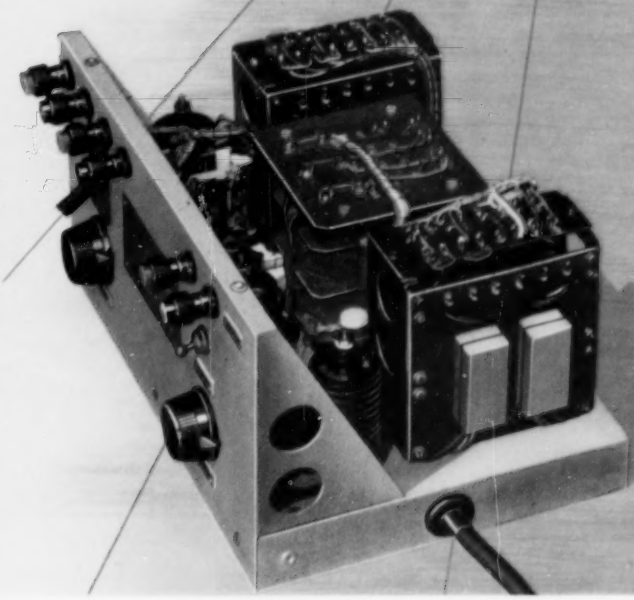
HIGH-STABILITY
Magnetic Amplifiers

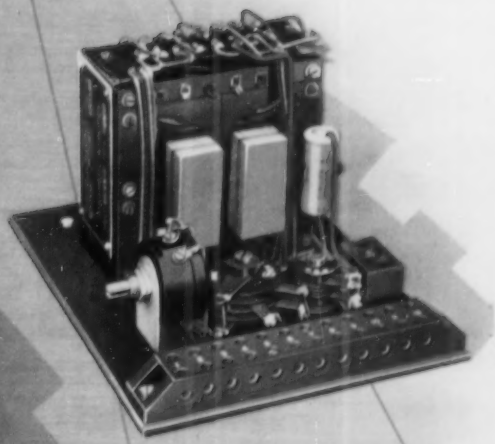


SINGLE-STAGE OR PUSH-PULL

An extensive range of standard types always available for prompt delivery

SPECIAL TYPES DESIGNED TO SUIT *YOUR* NEEDS





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Now FREE to all!

THE E-M TECHNICAL ADVISORY SERVICE

Regardless of whether your magnetic amplifier problem is simple or complex, the fact remains that the only reliable solution is that which entirely eliminates risk.

We therefore respectfully invite you to avail yourself of the wide resources of technical knowledge and practical experience possessed by the specialist technicians of our Magnetic Amplifier Division

Full technical data and illustrated leaflets promptly forwarded on request!

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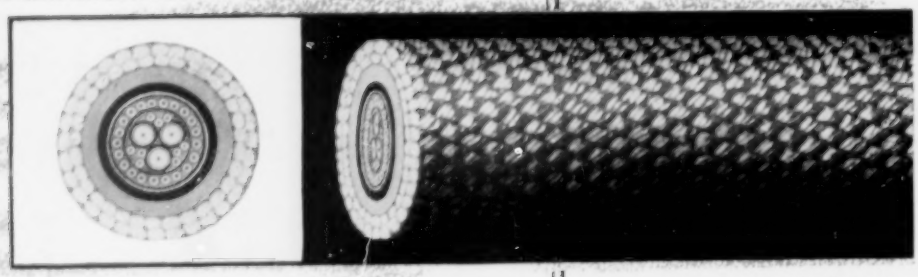


This cable carries weight

BICC Combined Camera Cable and Lifting Rope for underwater use.

This special multicore T/V camera cable is also used as a rope. It contains all the necessary electrical circuits and supports the full weight of the underwater T/V Camera by means of an overall hemp loom braid. It was designed and manufactured by BICC to meet the requirements of Messrs. Pye Ltd.

During recent trials the camera was repeatedly lowered and operated by this cable. Depths of 83 fathoms were reached and clear pictures obtained. Throughout the trials the cable was subjected to many severe tests.★ALL CIRCUITS REMAINED INTACT.

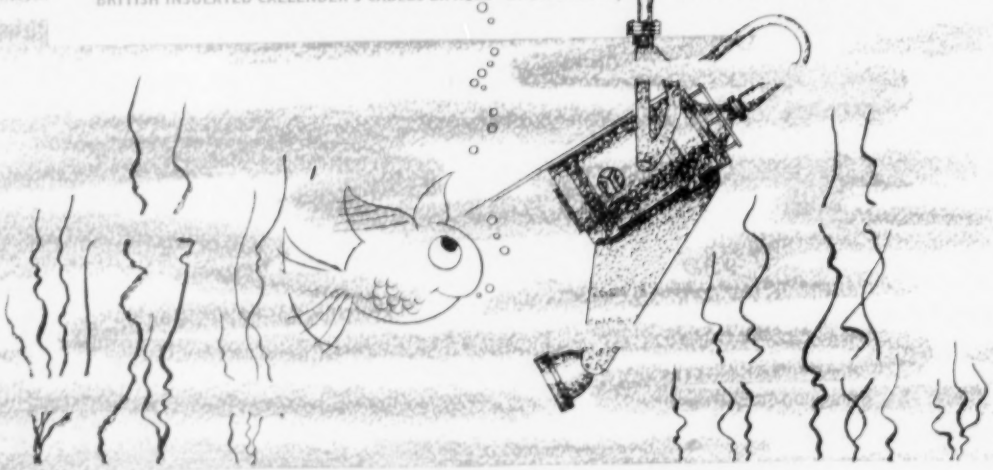
BICC ARE ALWAYS PREPARED TO DESIGN AND MANUFACTURE CABLES TO MEET SPECIAL REQUIREMENTS.

★ *It withstood a 2½ cwt. shock drop of 20 ft. out of water; towed the camera at speeds up to 12 knots; raised and lowered the camera at 250 ft. per minute over narrow diameter pulleys and capstan.*

Subsequent examination proved the cable and coupling to be completely waterproof, resistant to twisting and electrically intact.

BICC T/V CAMERA CABLES

BRITISH INSULATED CALLENDER'S CABLES LIMITED · 21 Bloomsbury Street · LONDON · WC1

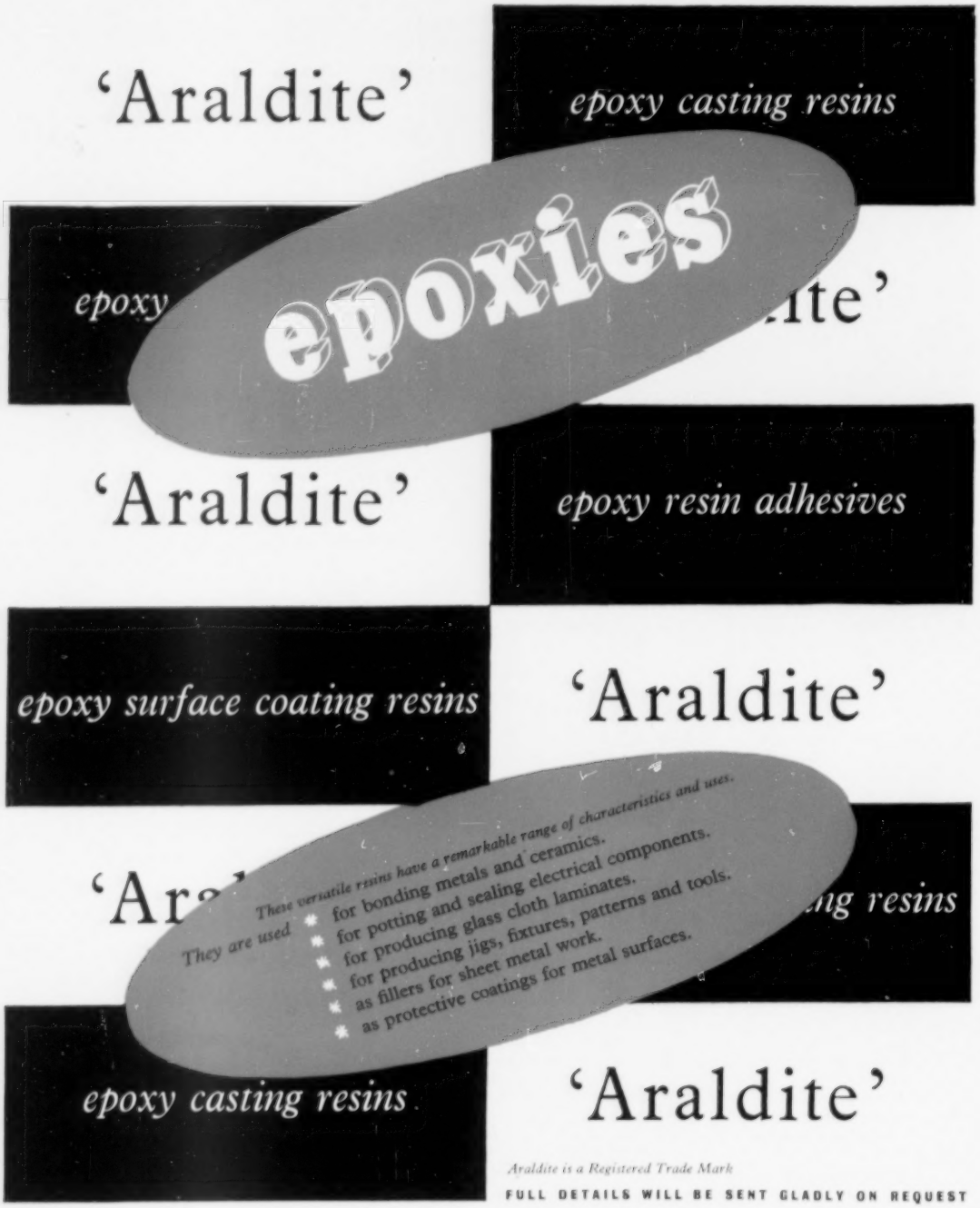


UNBRAKO SCREWS COST LESS THAN TROUBLE...

*F*rom every melt of incoming metal and through every stage of manufacture. Unbrako Screws are subjected to material, dimensional and physical laboratory tests that leave nothing to chance. You can use smaller screws or less screws when you specify Unbrako, that's why they are used in ever increasing quantities by industry throughout the world.

UNBRAKO

UNBRAKO SOCKET SCREW CO., LTD., COVENTRY



Araldite is a Registered Trade Mark

FULL DETAILS WILL BE SENT GLADLY ON REQUEST

Aero Research Limited

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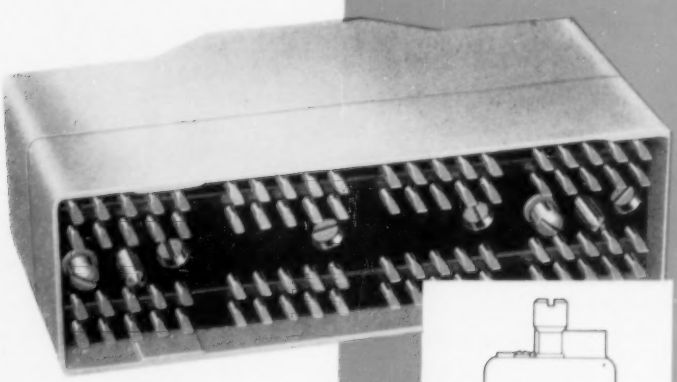
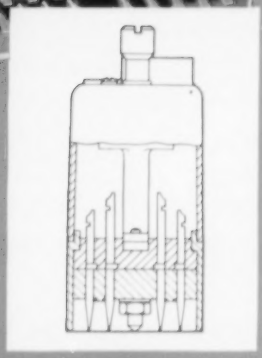
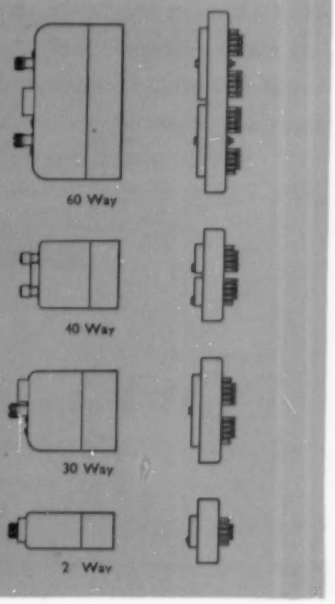
Designed with typical

Plessey *thoroughness*







A FLEXIBLE AND RELIABLE INTER-CONNECTION SYSTEM FOR MULTI-LINE CIRCUITS

A reliable, rapid and foo!-proof method of inter-connection for multi-line circuits has now been achieved by Plessey Design and Development through the introduction of a range of standardised Plugs and Sockets. This innovation has made possible a unit method of construction so superior in operation that it has become standard practice within the electronic and light electrical industries. Typical examples may be seen in television transmitters, radar assemblies, telephone exchanges and railway signalling systems.

This Publication is available on request . . . if you would like further information regarding these and other multi-way connectors, Plessy Publication No. 741 is available to manufacturers free on request.

BRIEF SPECIFICATION

Working volts—250 v. D.C. — Insulation resistance 100 megohms at 500 v. Maximum current —2 amps at 250 v. D.C. Contact resistance—1 milliohm. Temperature range +70°C to −40°C.

The plug is secured in the socket by a central clamping screw. Cables are firmly clamped to avoid strain. Plug contacts are chisel-type blades which tend to make the plugs self-ejecting from the socket on disengagement.

THE PLESSEY COMPANY LIMITED, ILFORD, ESSEX.

PR4a

Consistency of Performance

TRIAL BY STEAM

Sweltering in superheated steam may be some people's way of achieving perfection—Welwyn find this method ideal for *ensuring* it. For to test their hermetic sealing, Welmet metal film resistors undergo a Turkish Bath treatment in a pressure of two atmospheres. A drastic test indeed, but it proves beyond all doubt whether the vital gold-platinum alloy element is effectively protected. And it's part and parcel of the quality control that Welwyn practise with all their products.

MANUFACTURERS OF
WELWYN
ELECTRICAL
COMPONENTS

PROPERTIES OF THE WELMET INCLUDE:

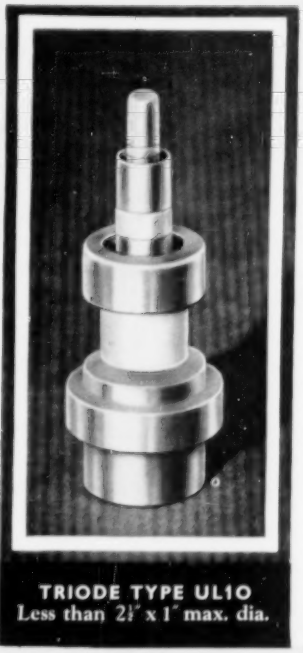
A load stability of 0·1%
No resistance change under tropical conditions
A rating of 1 watt at 70°C
A range of values from 1000Ω to 1MΩ
Selection tolerances of ±1%, ±2% and ±5%
The maximum voltage is defined only by the dissipation

WELWYN ELECTRICAL LABORATORIES LTD • BEDLINGTON • NORTHUMBERLAND

On Admiralty, Ministry of Supply (A.I.D. approved) and Post Office Lists

Now Available!

FERRANTI CERAMIC VALVES

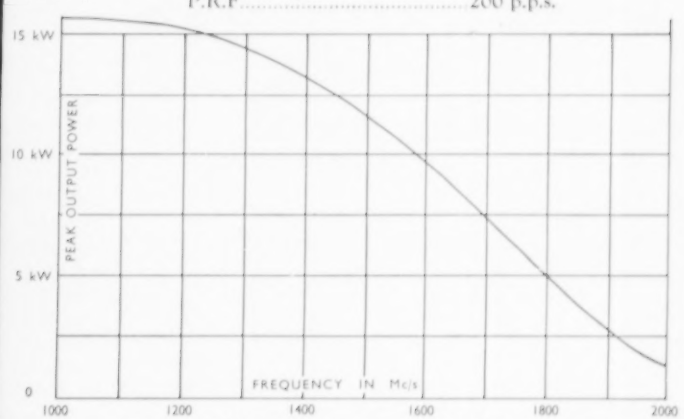


TRIODE TYPE UL10
Less than 2½" x 1" max. dia.

TRIODE TYPE UL10

RF Output Power under PULSE Conditions

Pulse V_a ..... 4 kV
Pulse Duration ..... 2½ μsec.
P.R.F ..... 200 p.p.s.



Also suitable for use as C.W. Oscillator or Amplifier giving outputs up to 15 Watts at frequencies up to 1000 Mc/s.

All Ferranti Ceramic Valves have the following outstanding advantages

- LONG LIFE
- REDUCED DIMENSIONS
- HIGH PEAK EMISSION
- HIGH MECHANICAL STRENGTH
- HIGH PERMISSIBLE TEMPERATURE OF OPERATION

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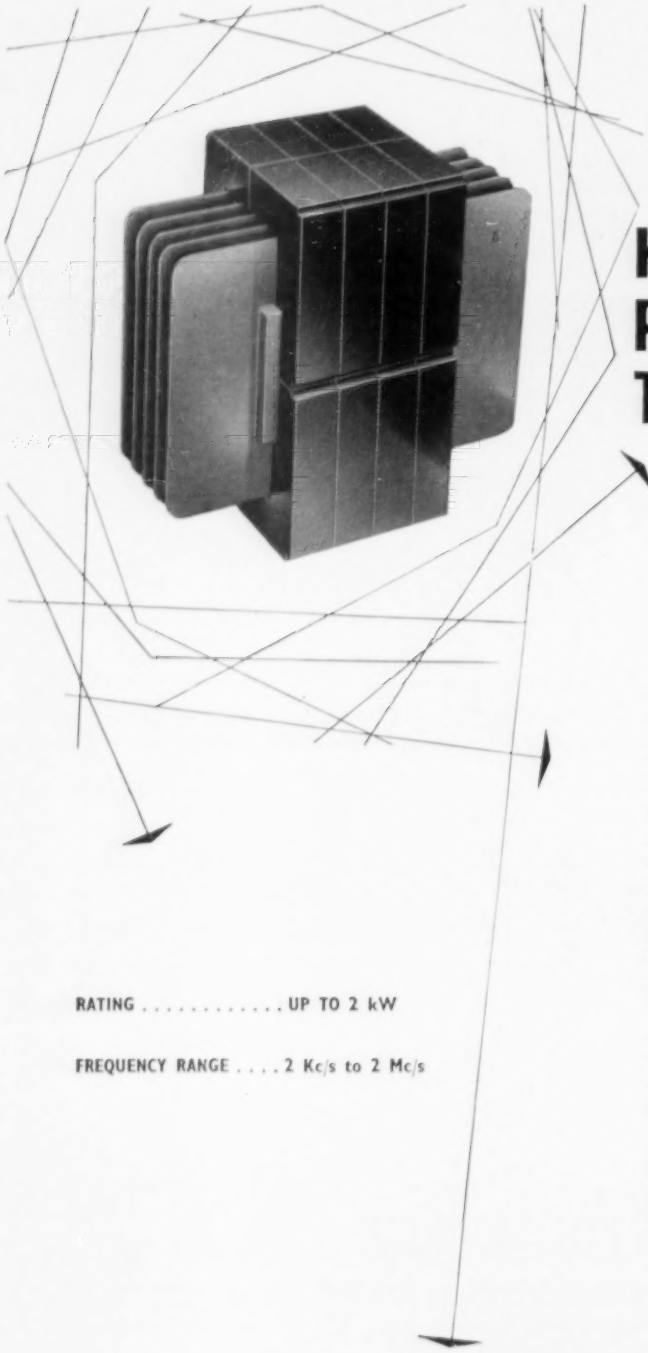

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WIRELESS ENGINEER

The Journal of Radio Research and Progress

ESTABLISHED 1923

Managing Editor: HUGH S. POCOCK, M.I.E.E. Editor: W. T. COCKING, M.I.E.E.

Technical Consultant: Professor G. W. O. HOWE, D.Sc., LL.D., M.I.E.E.

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Volume 33 · Number 4

APRIL 1956

CONTENTS

Editorial: New Rejector Circuit 77

Waveguide Filters
by M. H. N. Potok, B.Sc. 79

Clutter on Radar Displays
by J. Croney, B.Sc. 83

The Impedance Concept—Part 1
by C. G. Mayo and J. W. Head 96

Correspondence 103

New Books 104

Standard-Frequency Transmissions 104

Abstracts and References. Nos. 961-1268 .. .. A.73-A.94

Published on the fifth of each month

Annual Subscription: Home and overseas £2 9s. 0d.; Canada and U.S.A. $7.50

(Including Annual Index to Abstracts and References)

Published by Iliffe & Sons Ltd.

Editorial, Advertising and Publishing Offices: Dorset House, Stamford Street, London, S.E.1

Telephone: Waterloo 3333 (60 lines) · *Telegrams: Wirenger, Sedist, London*

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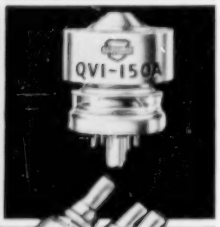

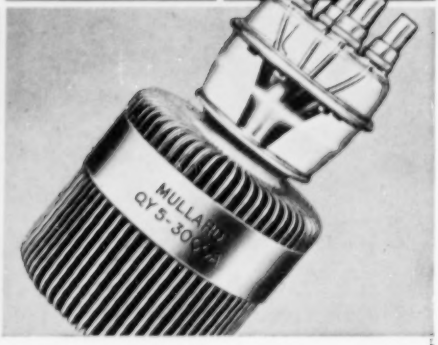

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MULLARD TYPE No.	AMERICAN TYPE No.	CV TYPE No.	DESCRIPTION	BASE	HEATER (V)	HEATER (A)	$V_{amax.}$ (V)	$p_{amax.}$ (W)	TYPICAL LOAD POWERS & FREQUENCIES (W)	TYPICAL LOAD POWERS & FREQUENCIES (Mc/s)
QV06-20	6146	CV3523	V.H.F. Power Tetrode	Octal	6.3	1.25	600	20	42 20	60 175
QQV03-10	6360	CV2798	V.H.F. Power Double Tetrode	B9A	6.3 12.6	0.83 0.42	300	2 x 5.0	14 11.5	100 200
QQV03-20A	6252	CV2799	V.H.F. Power Double Tetrode	B7A	6.3 12.6	1.3 0.65	600	2 x 10	39 15	200 600
QQV06-40A	5894	CV2797	V.H.F. Power Double Tetrode	B7A	6.3 12.6	1.8 0.9	750	2 x 20	72 45	200 500
QY3-65	4-65A	CV1905	V.H.F. Power Tetrode	B7A	6.0	3.5	3000	65	224 88	50 220
QY3-125	6155 4-125A	CV2130	V.H.F. Power Tetrode	B5F	5.0	6.5	3000	125	300 175	120 200
QY4-250	6156/4-250A	CV2131	V.H.F. Power Tetrode	B5F	5.0	14.1	4000	250	800 400	75 120
QV1-150A	4X-150A	CV2519	U.H.F. Power Tetrode	B8F	6.0	2.6	1250	150	156 112	165 500
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WIRELESS ENGINEER

Vol. 33 APRIL 1956 No. 4

New Rejector Circuit

ONE of the most difficult problems confronting the designer of a television receiver is that of securing adequate rejection of the sound signal without appreciably affecting the response of the video channel. In British television, and with the standard intermediate frequencies of 34·65 Mc/s for vision and 38·15 Mc/s for sound, the requirement is that the response of the vision channel should be at least 40 dB less at 38·15 Mc/s than it is at 37·65 Mc/s. Also, the response at this frequency, which is the upper edge of the video band, should not be down by more than a few decibels compared with that at lower frequencies.

The requisite sharp cut-off can only be obtained by the use of traps, absorbers, acceptors or rejectors, to quote the various names by which these circuits are known. Even then, very high Q in the trap circuit is desirable. Sometimes, bridged-T forms of trap are used; in these a phase balance is possible, so that the equivalent of a circuit of infinite Q can be obtained. Very high rejection is then possible, but the rejector bandwidth becomes exceedingly narrow.

An appreciable departure from the ideal conditions at the edge of the video band can occur in monochrome television without anything very drastic happening to the picture. The conditions for a colour picture are a good deal more stringent, however, for the phase of the colour components can be upset very easily by such defects.

A new form of sound-channel rejector was shown in the circuit of an experimental colour television receiver which has been recently described[1] and it has also been seen in some American colour-receiver circuit diagrams[2]. The overall response curve of the first set is reproduced from the first paper in Fig. 1 and it can be seen that the cut-off at the edge of the pass-band is almost incredibly sharp. Moreover, beyond the point of maximum attenuation, the rise in response is unusually small. These characteristics are largely due to a new form of rejector circuit, of which two are used in this receiver.

No published description of this rejector has yet appeared and it is understood to be due to

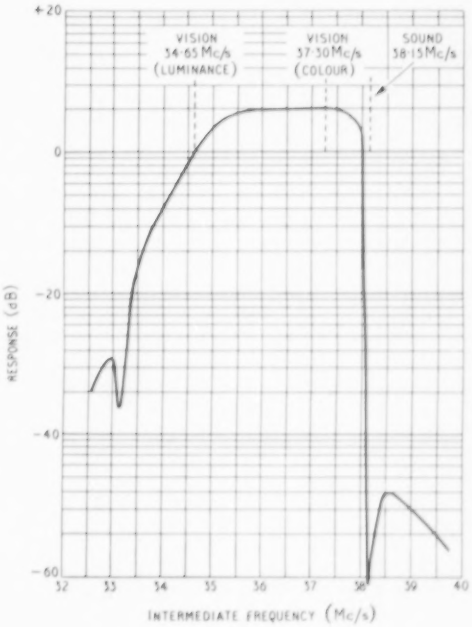


Fig. 1.

R.C.A. It is termed the bifilar-T trap and the essential part of the circuit is shown in Fig. 2. The trap proper is the conventional circuit L_1C_1 but it is connected into circuit via a centre-tapped coil of total inductance $4L$ having a resistance R across one of its windings. The two halves of this coil are bifilar wound to obtain maximum coupling between them.

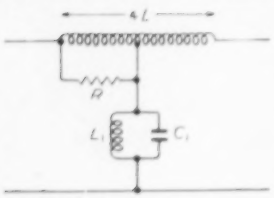
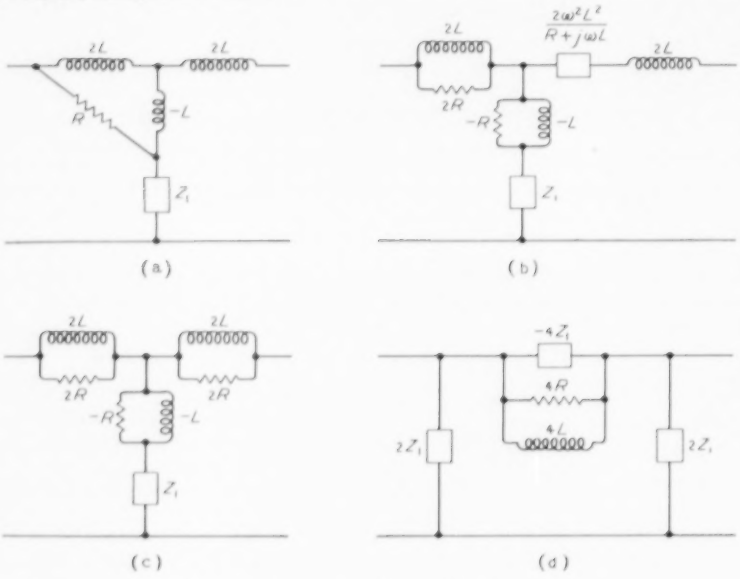


Fig. 2 (left).



Assuming that the coupling is, in fact, 100% and capacitance and resistance effects are negligible, an equivalent circuit has the form of Fig. 3(a) where Z_1 represents L_1C_1. By the application of the star-delta theorem to $2L$, R and $-L$, the circuit can be transformed to (b) and thence, by combining the elements of the right-hand series arm, to (c). The further use of the star-delta theorem enables the form (d) to be obtained.

If Z_1 is a circuit comprising L_1, C_1 and R_1 all in parallel, Fig. 3(d) can be re-drawn as in Fig. 4 and it is plain that the impedance of the series arm becomes infinite when

$$R = R_1$$

and

$$\omega^2 \frac{LL_1}{L - L_1} C_1 = 1$$

so that it is necessary to have $L > L_1$.

The network is a π of parallel-tuned circuits in which the losses of the circuit in the series arm are effectively zero. In addition to this, however, the series arm is peculiar in having both negative inductance and capacitance. Instead of being inductive below resonance and capacitive above like an ordinary tuned circuit, the circuit is inductive above resonance and capacitive below.

This is the property of the bifilar-T circuit which distinguishes it from other rejectors, like the bridged-T, which are also capable of providing infinite attenuation at a particular frequency.

In Fig. 4, each shunt arm has a frequency of parallel resonance given by $\omega^2 = 1/L_1C_1$. The series arm has a frequency of parallel resonance given by $\omega^2 = (1 - L_1/L)/L_1C_1$ and which is, therefore, lower. Considering the series arm and the right-hand shunt arm as a pair of parallel-resonance circuits in series, there is a series-resonance frequency given by $\omega^2 = (1 + L_1/L)/L_1C_1$ and is higher than either of the other two frequencies. Normally, with circuits having positive inductance and capacitance, this series-resonance frequency always lies between the two parallel-resonance frequencies.

Fig. 3 (left).

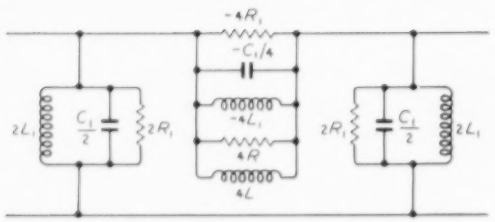


Fig. 4.

In the practical use of the bifilar-T circuit, other impedances are connected to the input and output terminals. The effective parallel-resonant frequencies of the shunt arms are, therefore, modified by such elements. The bifilar-T circuit of Fig. 2 may, for example, be used to couple together a pair of parallel-resonant circuits. In that case, the equivalent circuit for the whole

network takes the form of Fig. 5 where $L' < 2L_1 > L''$, $C' > C_1/2 < C''$ and $R' < 2R_1 > R''$. Alternatively, one or both of the terminating impedances may be simple shunt RC combinations, in which case the circuit remains unchanged but the inductance values are different.

In most practical cases, therefore, the complete

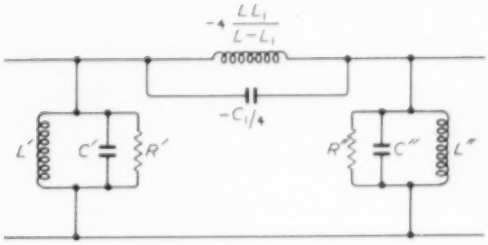


Fig. 5.

circuit of an intervalve coupling using a bifilar-T trap has the form of Fig. 5 and so is a pair of tuned circuits with 'top-end' coupling by a lossless trap. The difference between this circuit and others capable of infinite rejection is that the equivalent trap has negative inductance and capacitance.

A complete and exact analysis is too complex to undertake here but we feel it desirable to draw attention to some of the unusual features of this circuit. The equivalent circuits given here are not, of course, exact, because we have ignored the leakage inductance in the bifilar coil and also its self-capacitance. This is likely to be large, because of the bifilar winding.

W. T. C.

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[1] "Experimental Colour Receiver", by H. A. Fairhurst, *Wireless World*, March 1956, p. 112.

[2] "Circuitry Report on the Admiral Color TV Receiver", by John Schumacher, *Service*, March 1956, p. 14.

WAVEGUIDE FILTERS

Narrow Band with Quarter-Wave Transformers

By M. H. N. Potok, B.Sc., A.M.I.E.E., A.M.Brit., I.R.E.

(Department of Electrical Engineering, The Royal Technical College, Glasgow)

SUMMARY.—High-Q, low insertion-loss waveguide filters can be made comparatively easily by inserting into the guide a number of suitably spaced, quarter-wave long, dielectric sections.

1. Introduction

THE most common methods of obtaining high-Q waveguide transmission filters are either to couple the output to a high-Q cavity, or to introduce into the guide two inductive slits or apertures just less than half guide wavelength apart. The first method necessitates high precision and careful finish, while the second is very difficult to tune over any but a very narrow band. Both methods result in a high insertion loss[1].

Recent years have seen the development of reflection-type optical filters based on the principle of Fabry Perot interferometer. The method can be applied at microwave frequencies and Culshaw[2] reported some excellent results using for the two reflectors blocks consisting of sheets of polystyrene. The blocks were a quarter-wave thick and were separated by a quarter-wave in air from one another, their inner faces being kept a multiple of half a wavelength apart. His arrangement is reported to lead to a Q of 60,000 when the two blocks are a considerable distance apart but by its very nature it results in considerable insertion loss and is somewhat cumbersome to operate.

MS accepted by the Editor, June 1955

Very good results, in some way better than those obtained in air, can be got by placing the quarter-wave transformers in a waveguide. The following paragraphs develop a simple theory of such a filter based on the impedance and reflection concepts common in waveguide technique.

2. Analysis of Quarter-Wave Transformer Filters

Let λ_g and λ_0 be the waveguide and unbounded space wavelengths respectively when the medium is air, and λ_{gd} and λ_{0d} refer to these wavelengths when the medium is a dielectric of negligible loss and a dielectric constant κ times larger than that of air.

The wave impedance of a guide filled with air is

$$Z_a = \frac{120\pi\lambda_g}{\lambda_0} = \frac{120\pi}{\sqrt{1-\lambda_0^2/\lambda_c^2}}$$

while when the guide is filled with the dielectric

$$Z_d = \frac{120\pi\lambda_{gd}}{\lambda_{0d}} = \frac{120\pi}{\sqrt{\kappa-\lambda_0^2/\lambda_c^2}}$$

Since, as will be shown, the two types of transformers should have as different impedances as possible, one of them should have air as the medium since this gives the largest possible wave

impedance, while the other should have large permittivity κ.

For simplicity all the quantities referring to the air transformer will have a suffix 'a' while those referring to the dielectric will have suffix 'd'.

Let the ratio $\dfrac{Z_d}{Z_a}$ be denoted by $p = \dfrac{\sqrt{1-\lambda_0^2/\lambda_c^2}}{\sqrt{\kappa-\lambda_0^2/\lambda_c^2}}$

As an example, for polystyrene ($\kappa = 2{\cdot}56$) when $\lambda_0 = 3{\cdot}14$ cm and is propagated in the H_{10} mode through a standard X-band guide (0·9 in. × 0·4 in.); then $p = 0{\cdot}5$.

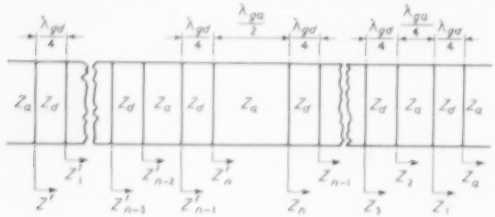

Fig. 1. Arrangement of filter.

Taking n quarter-wave transformers in a row so that the first is filled with a dielectric, second with air, etc., as in Fig. 1, the input impedance to the nth section (which is filled with dielectric), when the guide is matched at the output, is $Z_n = Z_d{}^{n+1}/Z_a{}^n = Z_a p^{n+1}$. The same impedance appears at the input to the preceding air-filled half-wave transformer (i.e., $Z_n{}' = Z_n$), hence, if p is small and n large, the half-wave transformer is an efficient cavity.

The frequency-selective nature of this arrangement can best be studied by assuming that λ_0 has been increased by a very small amount $\Delta\lambda$ to λ_1, then the total phase shift in a quarter-wave transformer (referred to λ_0) filled with dielectric d will be

$$\beta_{d1} l_d = \frac{2\pi\sqrt{\kappa\lambda_c^2-\lambda_1^2}}{\lambda_c\lambda_1}\cdot\frac{\lambda_c\lambda_0}{4\sqrt{\kappa\lambda_c^2-\lambda_a^2}}$$

$$\approx \frac{\pi}{2}\cdot\frac{\lambda_0}{\lambda_1}\cdot\left[1-\frac{\lambda_0\cdot\Delta\lambda}{(\kappa\lambda_c^2-\lambda_0^2)}\right]$$

and since $(\kappa\lambda_c^2-\lambda_0^2)$ is very likely to be of the same order of magnitude as $\lambda_0{}^2$ and for Qs in excess of 100, $\Delta\lambda$ will be less than $\lambda_0/100$

$$\beta_{d1} l_d = \frac{\pi}{2}\cdot\frac{\lambda_0}{\lambda_1} \text{ within } 1\%$$

and similarly for air

$$\beta_{a1} l_a = \frac{\pi}{2}\cdot\frac{\lambda_0}{\lambda_1}$$

Hence $\tan\beta_{d1} l_d \approx \tan\beta_{a1} l_a \approx \dfrac{1}{\pi/2-\pi/2.\lambda_0/\lambda_1}$

$$= \frac{2\lambda_1}{\pi\Delta\lambda} \approx \frac{2\lambda_0}{\pi\Delta\lambda}$$

Since this is likely to be a large quantity if $\Delta\lambda$ is small, it is more convenient to work with $x = \dfrac{\pi\Delta\lambda}{2\lambda_0}$

which is small, allowing one to neglect after inspection terms containing x^2 and higher powers of x.

Similarly one can analyse the effect of variation in λ_0 on wave impedances

$$Z_{d1} = \frac{120\pi_{de}}{\sqrt{\kappa\lambda_c{}^2-\lambda_0{}^2}} = \frac{120\pi\lambda_c}{\sqrt{\kappa\lambda_c{}^2-\lambda_0{}^2-2\lambda_0\Delta\lambda-\Delta\lambda^2}}$$

$$\approx Z_d\frac{1}{\sqrt{1-\dfrac{2\lambda_0\Delta\lambda}{\kappa\lambda_c{}^2-\lambda_0{}^2}}} \approx Z_d\left(1+\frac{\lambda_0\Delta\lambda}{\kappa\lambda_c{}^2-\lambda_0{}^2}\right)$$

$$\approx Z_d$$

In the same way, and provided $\Delta\lambda$ is very small $Z_{a1} = Z_a$. Thus the input impedance to the first dielectric transformer is

$$Z_1 = Z_d\frac{Z_a x + jZ_d}{Z_d x + jZ_a} \text{ at } x_1,$$

and by repeated application of this formula and neglecting terms containing x^2

$$Z_n = Z_d\frac{Z_a x(Z_a{}^n - Z_d{}^n) + jZ_d{}^n(Z_a - Z_d)}{Z_d x(Z_a{}^n - Z_d{}^n) + jZ_a{}^n(Z_a - Z_d)}$$

$$= Z_a\frac{px(1-p^n) + jp^{n+1}(1-p)}{px(1-p^n) + j(1-p)}$$

which simplifies to

$$Z_a\left(p^{n+1} - j\frac{px}{1-p}\right) \text{ if } p^n \ll 1.$$

Using the same procedure as that outlined in connection with the quarter-wave transformer the phase shift across a half-wave transformer at λ_1 becomes $= \pi\lambda_0/\lambda_1$ and the tangent of this phase shift becomes $= -2x$.

Thus if Z_n is the input impedance to the nth quarter-wave transformer, which is dielectric filled, the input impedance to the air-filled half-wave transformer preceding Z_n is $Z_n{}'$

$$Z_n{}' = Z_a\frac{Z_n - jZ_a 2x}{Z_a - jZ_n 2x}$$

$$= Z_a\frac{[p(1-p^n) + 2(1-p)]x + jp^{n+1}(1-p)}{[p(1-p^n) + 2p^{n+1}(1-p)]x + j(1-p)}$$

$$\approx Z_a\left(p^{n+1} - j\frac{2-p}{1-p}x\right)$$

If this section is now preceded by an identical set of quarter-wave transformers to that following the half-wave section, the input impedance to the whole arrangement will be at λ_1

$$Z' = Z_a\frac{Z_n{}' px(1-p^n) + jZ_a p^{n+1}(1-p)}{Z_a px(1-p^n) + jZ_n{}'(1-p)}$$

$$= Z_a\frac{xp^{n+2} + jp^{n+1}(1-p)}{2x + jp^{n+1}(1-p)}$$

Since the reflection coefficient $\rho = \dfrac{Z' - Z_a}{Z' + Z_a}$ and at half-power frequency $|\rho|^2 = \frac{1}{2}$ one obtains

$$|\rho|^2 = \frac{4x^2}{4x^2 + 4p^{2n+2}(1-p)^2} = \frac{1}{2}$$

hence

$$x^2 = p^{2n+2}(1-p)^2$$

therefore

$$x = p^{n+1}(1-p)$$

Since

$$x = \frac{\pi\Delta\lambda}{2\lambda_0} = \frac{\pi}{4Q}$$

i.e., $Q = \dfrac{\pi}{4p^{n+1}(1-p)}$

If one follows the previous example since

$$p = 0{\cdot}5,\ Q = \frac{\pi}{4 \times 0{\cdot}5^{n+1} \times 0{\cdot}5} = 2^n\pi$$

leading to Table 1.

TABLE 1

n	7	9	11	13	15
Q	400	1,600	6,400	25,600	102,400

A further substantial increase in Q can be obtained by making the centre section equal to an integral multiple of half-wavelengths. When the number of half-wavelengths is m, it can be shown by following the earlier procedure that

$$Q = \frac{\pi[m(1-p) + p]}{4p^{n+1}(1-p)}$$

hence if

$$p = 0{\cdot}5,\ Q = \frac{\pi(m+1)}{4p^{n+1}} = 2^{n-1}\pi(m+1)$$

Consider now the effect of an error in the lengths of the transformers such that the phase shift over each transformer is less than $\pi/2$ by a small angle x_1 so, that at λ_0 the input impedance

$$Z_n = Z_a\left(p^{n+1} - j\frac{px_1}{1-p}\right)$$

Reference to a Smith's Chart will make it clear that if the input impedance to the whole arrangement is to be $Z' = Z_a$ then

$$Z'_n = Z_a\left(p^{n+1} + j\frac{px_1}{1-p}\right)$$

hence the half-wave transformer section must be made longer than π by a small angle $\dfrac{2px_1}{1-p}$. Let now λ_0 increase to λ_1, then the phase shift across each quarter-wave transformer will be further reduced by a small angle x_2 while the phase shift across the half-wave transformer is reduced by $2x_2$.

Hence at λ_1

$$Z_n = Z_a\left[p^{n+1} - j\frac{p(x_1+x_2)}{1-p}\right]$$

$$Z_n' = Z_a\frac{Z_n + jZ_a\left(2x_1\dfrac{p}{1-p} - 2x_2\right)}{Z_a + jZ_n\left(2x_1\dfrac{p}{1-p} - 2x_2\right)}$$

$$= Z_a\frac{p^{n+1} - j\dfrac{p(x_1+x_2)}{1-p} + j\dfrac{2}{1-p}(x_1p + x_2p - x_2)}{1 + j\left(p^{n+1} - j\dfrac{px_1 + px_2}{1-p}\right)\dfrac{2}{1-p}(x_1p + x_2p - x_2)}$$

$$\approx Z_a\left[p^{n+1} + j\frac{p(x_1+x_2) - 2x_2}{1-p}\right]\left[1 - j\frac{2p^{n+1}}{1-p}(px_1 + px_2 - x_2)\right]$$

$$\approx Z_a\left[p^{n+1} + j\frac{p(x_1+x_2) - 2x_2}{1-p}\right]$$

and

$$Z' = Z_a\frac{Z_n'p(x_1+x_2) + jZ_ap^{n+1}(1-p)}{Z_ap(x_1+x_2) + jZ_n'(1-p)}$$

$$= Z_a\frac{p^{n+2}(x_1+x_2) + jp^{n+1}(1-p)}{2x_2 + jp^{n+2}(1-p)}$$

$$\approx Z_a\frac{p^{2n+2}(1-p)^2 + jp^{n+1}(1-p)2x_2}{4x_2^2 + p^{2n+2}(1-p)^2}$$

which is seen to be of the same form as original Z', hence it follows that provided the error is very small there will be no reduction in Q.

The same argument can be used to prove that the arrangement can be tuned over a narrow range of frequencies by varying the length of the central half-wave transformer.

The above discussion assumes no losses in dielectric and waveguide. In practice, such a condition is unobtainable and the actual Q values will be considerably reduced. To check the results, some simple experiments were carried out.

3. Experimental Evidence

The quarter-wave transformers were made out of polystyrene sheet machined down to 0·214 in. corresponding to quarter-wavelength in polystyrene ($\kappa = 2{\cdot}56$) in a 0·9-in. × 0·4-in. guide when $\lambda_0 = 3{\cdot}14$ cm ($f = 9{,}550$ Mc/s). The sheet was cut up into small sections fitting closely into the guide, the sections being held together 0·425-in. apart ($\lambda_g/4$ in air-filled guide) by two very thin strips of polystyrene. The drawn-copper waveguide had a narrow slit in the top

face so that the separation between the two sets of transformers could be adjusted to the desired frequency. The graph of Fig. 2 gives the plot of log Q against the number of polystyrene sections used ($= n + 1$ in the previous notation) when the centre transformer is half-wave long. Since the theoretical formula assumes $Q \ll 100$ the values corresponding to four and six polystyrene sections have been calculated directly step by step. It is seen that the experimental curve is considerably below the theoretical and it appears to tend to a limit around $Q = 10{,}000$. Lack of suitable equipment did not allow the extension of the experiment beyond 14 sections. The overall length of the filter was $9\frac{1}{2}$ in.

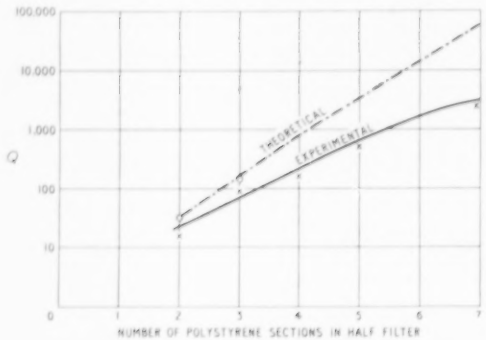

Fig. 2. *Theoretical (assuming no losses) and experimental relation between Q of filter and number of dielectric sections.*

In addition, measurements were taken with 14 polystyrene sections at wavelengths outside 3·14 cm giving the results of Table 2.

TABLE 2

f	9,407	9,547	9,602 Mc/s
Q	2,350	2,700	2,300

This shows that Q does tend to fall off below and above design-centre frequency, nevertheless the drop is not excessive. By doubling the length of the centre transformer from half to full wavelength the Q was found to increase from 2,700 to 3,400, an increase of 26% in place of the 50% estimated theoretically. The differences may be accounted by losses in polystyrene and guide walls. The insertion loss of the filter was less than 2 dB.

4. Conclusions

It has been shown that high-Q low-loss transmission filters can be obtained by the comparatively simple method of building up a chain of alternately high and low quarter-wave transformers. The arrangement is tunable over a narrow range of frequencies around the design centre.

The Q estimated on the basis of no losses is given by

$$Q = \frac{\pi}{4} \cdot \frac{m(1 - p) + p}{p^{n+1}(1 - p)}$$

where

p = ratio of dielectric to air wave impedances
$n + 1$ is the total number of dielectric sections used in the whole of the filter
m = number of half-wave air transformers in the centre of the filter

Since Q increases as p decreases, p must be as small as possible and the use of waveguide helps to reduce it appreciably.

If one considers losses in dielectrics and guide walls then the resultant Q values will be much reduced but they are still found to be large and in fact considerably larger than those obtainable with an identical set-up in air or in a coaxial cable of uniform structure (in air, the Q of a 14-polystyrene-section filter with half-wave transformer at centre and assuming no losses, is 1,550 compared with the experimentally obtained 2,700 in a waveguide).

Further work is contemplated; in particular, to study the effect of low-loss waveguides and higher permittivity low-loss dielectrics, such as quartz.

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CLUTTER ON RADAR DISPLAYS

Reduction by use of Logarithmic Receivers

By J. Croney, B.Sc.

(Royal Naval Scientific Service, Admiralty Signal and Radar Establishment)

SUMMARY.—An analysis is developed of the action of an idealized logarithmic receiver followed by a differentiating circuit (high-pass filter), upon inherent receiver noise, sea-clutter and rain-clutter echoes. The analysis is extended to estimate the extent to which the performance of the practical logarithmic receiver may depart from that of the idealized receiver. Results are given of experiments with logarithmic receivers on both S and X band radars. The loss which occurs when a differentiating circuit follows a logarithmic receiver is stated, the cause examined and a method of minimizing the loss suggested. The important design parameters of a logarithmic receiver for clutter reduction are dealt with.

1. Introduction

THE investigations set forth in this paper have been carried out by the author, using both S and X band radars at different times in recent years. The objective has been to examine the effectiveness of logarithmic receivers, followed by differentiating circuits (high-pass filters), in reducing the intensity of sea and rain clutter interference on radar displays[2] to that of the normal inherent noise background. A theoretical treatment of the subject is given, which is followed by the collected results of the various experiments. Some of the effects met with, and the conditions which must be observed for a worth-while measure of success, were at first difficult to understand. As the work proceeded a reasonably clear picture of the action of the logarithmic receiver and differentiating circuit on noise and clutter did emerge, and enabled the correct parameters for these and the video amplifier feeding the display to be established.

Against sea clutter the most that can be expected from theoretical considerations of the logarithmic receiver and differentiating circuit is that it should be as effective as a so-called linear receiver followed by a perfectly-adjusted swept-gain or sensitivity-time control (s.t.c.). It should be noted here that the perfectly-adjusted swept-gain control is an ideal which cannot be realized in practice even with a highly-skilled operator. In the first place, the waveform of the control voltage, which must be applied to give perfect suppression of the clutter envelope to noise level, cannot be attained practically even for a single restricted arc of azimuth and, since the clutter envelope varies significantly with azimuth, only a compromise setting of the swept-gain control can be used for the whole three hundred and sixty degrees of azimuth. The logarithmic receiver may in theory, therefore, be expected to achieve automatically a little more than even a skilled operator can produce with the swept-gain control. Against rain clutter the swept-gain principle cannot be applied, and the only existing measure in Civil Marine Radars is to follow the linear receiver by a differentiating or fast time-constant circuit (f.t.c.). This has the effect of reducing the intensity of the clutter background and enabling large targets to be seen within it. It is expected that by similar use of a logarithmic receiver the rain clutter background would be rendered indistinguishable from the inherent noise.

MS accepted by the Editor, May 1955

Fig. 1 (above). *Noise along a single range trace.*

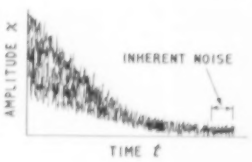


Fig. 2 (right). *Sea clutter along single range trace.*

2. Properties of Receiver Noise and of Sea Clutter

Fig. 1 represents receiver noise, and Fig. 2 sea clutter, along a single range trace of an A-scope display. The ordinate scale represents amplitude (x), and the abscissae range, which is proportional to time (t).

2.1. *Properties along a Single Range Trace*

The properties of noise and sea clutter will first be considered along any single range trace. Let the r.m.s. value of the amplitude x be represented by σ_0. Considering noise first, we have that the intensity is roughly constant (i.e., $\sigma_0 =$ constant); that the auto-correlation in amplitude extends over an interval approximately equal to $1/B$ where B is the receiver bandwidth; and that the amplitude x is distributed according to a Rayleigh probability distribution.

Taking now the case of sea clutter; there is a general decrease in intensity of clutter with

increasing range (i.e., σ_0 falls); the auto-correlation extends over an interval T (where T = radar pulse length); and the amplitude x is not distributed according to a Rayleigh distribution.

2.2. *Properties 'across' a Sequence of Range Traces at any Fixed Range*

The properties are now considered at a fixed element of range from one range trace to the next and so on. This may be considered as moving 'across' or perpendicular to the range traces. In the case of noise, we have that the intensity is again constant (i.e., σ_0 = constant); there is no correlation from trace to trace; and that the amplitude x is distributed according to a Rayleigh probability distribution.

In the case of sea clutter, the intensity is roughly constant (i.e., σ_0 = constant); the correlation extends over a finite number of traces for a stationary aerial and may extend over the beam-width of an aerial for a rotating aerial; and the amplitude x is distributed roughly according to a Rayleigh probability distribution.

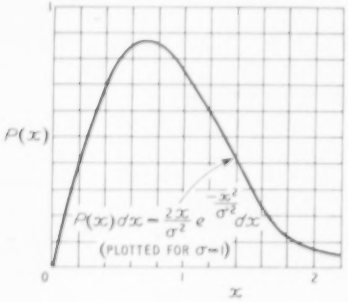


Fig. 3. *Curve of Rayleigh distribution of x plotted for scale factor $\sigma = 1$.*

2.3. *Differences between Noise, Sea Clutter and Rain Clutter*

From the foregoing, the important differences between noise and sea clutter are twofold. First, there is the variation in intensity of sea clutter along a range trace, which will result in saturation of a p.p.i. at short ranges, if the biasing and limiting levels are adjusted for long ranges where only noise is present. Secondly, the grain size of a sea-clutter background is usually coarser than that of a noise background, due to the trace-to-trace correlation of clutter, and any excess of T over $1/B$.

Given equality between T and $1/B$ the only observable difference between noise and rain clutter lies in the variation of intensity of rain clutter with range. This can take any form depending upon the variation of rainfall rate within the storm area.

The object of using a logarithmic receiver followed by a differentiating circuit is to eliminate this variation in intensity with range of both sea and rain clutter.

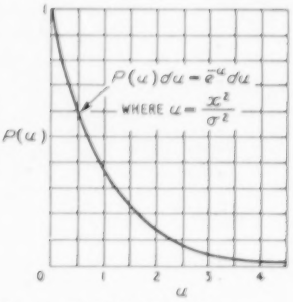


Fig. 4. *General curve representing Rayleigh distributions of x for all scale factors σ.*

3. Theory of the Logarithmic Receiver

3.1. *The Idealized Logarithmic Receiver*

It has been stated above that the amplitudes of inherent receiver noise, sea clutter, and rain clutter all obey more or less a Rayleigh probability distribution (after detection) when measured at a fixed range from trace to trace.

The Rayleigh probability distribution may be written as:

$$P(x)\,dx = (2x/\sigma^2)\exp.(-x^2/\sigma^2)\,dx,\quad x > 0 \qquad (1)$$

In this equation, x is the instantaneous amplitude and σ is the r.m.s. fluctuation about the mean (or scale factor).

By substituting a new variable:

$$u = x^2/\sigma^2,\quad u > 0 \qquad (2)$$

the distribution can be transformed to:

$$P(u)\,du = \exp.(-u)\,du,\quad u > 0 \qquad (3)$$

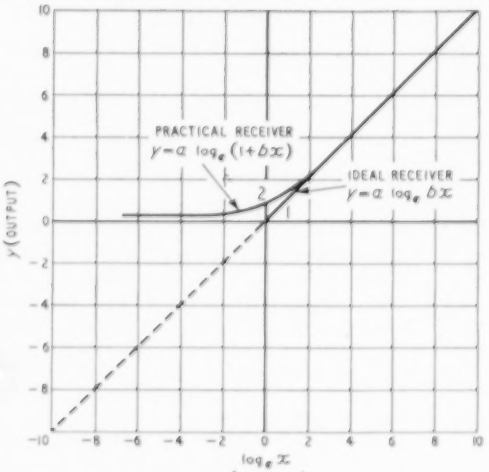


Fig. 5. *Curves representing laws for the ideal and practical logarithmic receivers. Constants 'a' and 'b' taken as unity for convenience in plotting.*

The original distribution (Equ. 1) is shown in Fig. 3 and the transformed distribution (Equ. 3) is shown in Fig. 4. It may be noted in passing that Equ. (3) is independent of σ, and Fig. 4 therefore represents all Rayleigh distributions regardless of the scale factor, σ.

We now consider an idealized logarithmic amplifier in which the input, x, is related to the output, y, by the law:

$$y = a \log_e(bx) \quad .. \quad .. \quad .. \quad (4)$$

where a and b are constants of the amplifier. This law is shown graphically by curve (1) of Fig. 5, the constants a and b being assumed unity for convenience of plotting. It can be seen that as the input falls below unity ($x < 1$, $\log x < 0$) the output assumes negative values down to minus infinity (dotted part of characteristic).

Equ. (4) may be written as:

$$y = (a/2) \log_e(b^2x^2) \quad .. \quad .. \quad .. \quad (5)$$

which, by substitution for x from Equ. (2), becomes:

$$y = (a/2) \{\log_e(b^2\sigma^2) + \log_e u\} \quad .. \quad (6)$$

If a random input signal of amplitude distribution $P(u)\,du$ (Equ. 3) is applied to the receiver input, the resulting probability distribution, $P(y)\,dy$, at the output of the amplifier will have the following parameters:

Mean value

$$= E(y) = \int_0^\infty (a/2)[\log b^2\sigma^2 + \log u]\, P(u)\, du \quad (7)$$

$$= (a/2) \left\{\log b^2\sigma^2 + \int_0^\infty (\log u)\, P(u)\, du\right\} \quad .. \quad .. \quad .. \quad (8)$$

since $\int_0^\infty P(u)\, du = 1$ from probability definitions.

Similarly the mean-squared value of y, $E(y^2)$ is given by:

$$E(y^2) = \int_0^\infty (a^2/4)\left[\log b^2\sigma^2 + \log u\right]^2 P(u)\, du$$

$$= (a^2/4) \left\{(\log b^2\sigma^2)^2 + 2 \log b^2\sigma^2 \int_0^\infty (\log u) P(u)\, du + \int_0^\infty (\log u)^2\, P(u)\, du\right\} \quad .. \quad (9)$$

Hence the variance of y is given by:

$$E(y^2) - \{E(y)\}^2 = (a^2/4) \left\{\int_0^\infty (\log u\,)^2 P(u)\, du - \left[\int_0^\infty (\log u)\, P(u)\, du\right]^2\right\} \quad .. \quad .. \quad .. \quad (10)$$

Inserting the value of $P(u)\,du$, given for all Rayleigh distributions by Equ. (3) as exp. $(-u)\,du$, we have:

$$E(y^2) - \{E(y)\}^2 = (a^2/4) \left\{\int_0^\infty (\log u)^2 \exp.(-u)\, du - \left[\int_0^\infty (\log u) \exp.(-u)\, du\right]^2\right\} \quad .. \quad .. \quad (11)$$

The integrals can be found in tables of Laplace transforms, and Equ. (11) then reduces to:

$$E(y^2) - \{E(y)\}^2 = (a^2/4).(\pi^2/6) \quad .. \quad (12)$$

We have, therefore, the result that if any Rayleigh distribution is applied to the input of a receiver with an idealized logarithmic input–output law, the resulting r.m.s. fluctuation about the mean at the receiver output is constant, independent of the r.m.s. amplitude of the input fluctuations, and given by $(a\pi)/(24)^{\frac{1}{2}}$ where a is the slope of the logarithmic law. Fig. 6 illustrates this constant r.m.s. fluctuation of the clutter and noise signals about a varying mean level at the output of the logarithmic receiver, and in Fig. 7 the mean level has been removed by a high-pass filter, leaving the clutter and noise fluctuations at a constant r.m.s. amplitude across the display trace.

Fig. 6 (left). *Clutter output from logarithmic receiver before passage through high-pass filter.*

Fig. 7 (right). *Clutter output after removal of mean level by high-pass filter.*

3.2. *Practical Logarithmic Receiver*

A receiver with an idealized logarithmic characteristic (curve 1, Fig. 5) is clearly unattainable in practice; the practical logarithmic receiver having a law given by:

$$y = a \log_e (1 + bx) \quad .. \quad .. \quad (13)$$

shown as curve 2 in Fig. 5.

As the value of the input x becomes small, the law of the practical receiver begins to depart significantly from that of the ideal receiver; the output y of the practical receiver being zero for zero input, and the corresponding output of the ideal receiver being minus infinity. The above analysis based upon the ideal logarithmic law will therefore involve some error, the lower limits of the integrals in Equ. (11) corresponding to outputs of minus infinity which are impossible with the practical receiver. Attempts at an exact analysis for the practical receiver using Equ. (13) rapidly lead to intractable mathematics. However, the error resulting from the use of the idealized law will depend upon the extent to which the negative-going outputs of the ideal receiver contribute to the result of Equ. (12).

This error can be estimated in the following way.

Equ. (4) gives negative values to y when $bx < 1$ and, to avoid this, we define a new variable Y as follows:

$$\begin{aligned} Y &= a \log (bx) \text{ when } bx \geq 1 \\ Y &= 0 \quad\quad \text{when } bx < 1 \end{aligned} \quad \ldots \quad (14)$$

Let $u = z$ when $bx = 1$. From Equ. (2) $u = x^2/\sigma^2$, therefore:

$$z = 1/(b^2\sigma^2) \text{ or } b^2\sigma^2 z = 1 \quad \ldots \quad \ldots \quad (15)$$

Now

$$E(Y) = \int_z^\infty (a/2) \log (b^2\sigma^2 u)\, P(u)\, du$$

$$= E(y) - \int_0^z (a/2) \log (b^2\sigma^2 u)\, P(u)\, du \quad (16)$$

Similarly

$$E(Y^2) = E(y^2) - \int_0^z (a^2/4)(\log b^2\sigma^2 u)^2\, P(u)\, du \quad (17)$$

When z is sufficiently small exp. $(-u) \approx 1 - u$, $(0 < u < z)$. Also the upper limit of u in the integrals of Equs. (16) and (17) is given by

$$z = 1/(b^2\sigma^2) = 0{\cdot}01$$

(assuming that the r.m.s. fluctuation of noise extends over 20 dB of the receiver characteristic). We can thus put exp. $(-u) \approx 1{\cdot}0$ as a first approximation, and therefore $P(u)$ can be written as unity in Equs. (16) and (17). Doing this, and changing the variable to $v = b^2\sigma^2 u$, we get:

$$\begin{aligned} &(a/2) \int_0^z \log (b^2\sigma^2 u)\, P(u)\, du \\ &= (a/2b^2\sigma^2) \int_0^1 \log v\, dv \\ &= (a/2b^2\sigma^2) \Big[v\,(\log v - 1) \Big]_0^1 \\ &= -(a/2b^2\sigma^2) \quad \ldots \quad (18) \end{aligned}$$

and

$$\begin{aligned} &(a^2/4) \int_0^z (\log b^2\sigma^2 u)^2\, P(u)\, du \\ &= (a^2/4b^2\sigma^2) \int_0^1 (\log v)^2\, dv \\ &= (a^2/4b^2\sigma^2) \Big[v\,(\log v - 1)^2 + v \Big]_0^1 \\ &= (2a^2/4b^2\sigma^2) \quad \ldots \quad (19) \end{aligned}$$

Substituting these results in Equs. (16) and (17) we get:

$$\begin{aligned} &\text{Variance of } Y = E(Y^2) - \{E(Y)\}^2 \\ &= E(y^2) - \{E(y)\}^2 - (a^2/2b^2\sigma^2) - (a/b^2\sigma^2)\, E(y) \\ &\qquad - (a/2b^2\sigma^2)^2 \quad \ldots \quad \ldots \quad (20) \end{aligned}$$

$$= (a^2/4)\, \{\pi^2/6 - (2/b^2\sigma^2)\, [1 + \log b^2\sigma^2 - \gamma + 1/(2b^2\sigma^2)]\} \quad (21)$$

the integrals for $E(y)$ and $E(y^2)$ being found in tables of Laplace transforms. (γ = Euler's constant $\approx 0{\cdot}577$.)

The correction factor to be subtracted from the multiplying constant, $(\pi^2/6)$, in Equ. (12) because of the inability of the practical receiver to allow negative outputs, is therefore given by:

$$(2/b^2\sigma^2)[1 - \gamma + 1/(2b^2\sigma^2) + \log b^2\sigma^2] \quad \ldots \quad (22)$$

If the receiver is logarithmic to about 20 dB below r.m.s. noise level, we can take $b\sigma = 10$ for noise only, and the correction term will be of magnitude 0·1.

For clutter signals which may be as great as 80 dB above r.m.s. noise, σ^2 is greater by a factor of 10^8 and the correction term then becomes negligible. It is thus concluded that, as the noise plus clutter varies over a range of 80 dB, the variance of the output Y will vary from $(a^2/4)[\pi^2/6 - 0{\cdot}1]$ to $(a^2/4).(\pi^2/6)$, that is about 1 part in $16{\cdot}5 \approx 0{\cdot}25$ dB. Hence the variance of Y can be taken as sensibly independent of σ.

This result applies to a receiver having an ideal logarithmic law from zero output upwards, and in which the input r.m.s. noise fluctuations extend over the first 20 dB of the logarithmic characteristic. The full part of curve 1 in Fig. 5 shows the characteristic, and the curve for the practical receiver (curve 2, Fig. 5) departs from the desired characteristic for low values of the input x. However, by designing the practical receiver so that the constant b (gain) is very large, all the noise fluctuations within minus 20 dB of the r.m.s. noise level (99%) can be sensibly lifted on to the ideal characteristic (curve 1), and the above result is then closely applicable.

Before leaving these analytical considerations, it is worth noting that Equ. (10) is quite general for the logarithmic receiver and that if we insert for $P(u)\, du$ any distribution function of u which is independent of σ the resulting variance will be a constant. In practice, this means that the logarithmic receiver would be equally effective against any randomly varying input signal in which the amplitude x fluctuates over *positive* values according to a type of distribution in which the probability of x lying between x and $x + dx$ is determined by a scale factor σ; i.e.,

$$P(x)\, dx = P^*(x/\sigma)\, dx/\sigma, \quad x > 0.$$

Such a distribution can be transformed by the substitution $u = x^2/\sigma^2$ to a distribution of the type $P(u)\, du$. As shown in Equs. (1) to (3), the Rayleigh distribution is one example of such a probability function; there may be others.

4. Experiments with Logarithmic Receivers

4.1. *Limits of the Logarithmic Input–Output Law, and their Achievement*

It has been shown in Section 3 that a logarithmic receiver followed by differentiation can in theory reduce the r.m.s. clutter level to the r.m.s. level of inherent noise, provided that the logarithmic law extends at its upper limit to the highest level of clutter fluctuations (usually not more than 80 dB above r.m.s. noise) and at its lower limit to the deep troughs of the clutter and noise fluctuations. The extent to which the receiver should remain logarithmic below r.m.s.

noise level is of cardinal importance, and it can be stated at once that if the input–output characteristic ceases to be logarithmic at the r.m.s. noise level, little or no reduction in the intensity of the clutter background on a p.p.i. will be obtained in comparison with a linear receiver. However, as the logarithmic law is progressively extended below r.m.s. noise level, the disparity in intensity between the clutter background and the noise background is at first rapidly reduced, and then more slowly, until the logarithmic characteristic has been extended to a depth of about 20 dB below r.m.s. noise, beyond which no noticeable improvement appears to be realized. This is understandable since the probability of any fluctuation of the Rayleigh distribution reaching a value less than 20 dB below the r.m.s. level is only about 1%.

It is essential therefore to maintain the logarithmic characteristic uniformly down to a level of approximately 20 dB below the r.m.s. noise level. It is inadvisable to maintain the logarithmic characteristic to levels lower than 20 dB below r.m.s. noise, since nothing perceptible is gained in clutter reduction, and the extent to which the receiver is logarithmic above r.m.s. noise level is likely to be reduced as a consequence. This could be serious if the upper limit of the clutter fluctuation then lay beyond the upper limit of the logarithmic range of the receiver.

Methods of achieving a logarithmic response are covered by references one, two and three. All the experiments here reported were conducted with receivers constructed by the method of reference three. Certain important aspects of the design parameters covered in Section 5 are intended to supplement the material of reference three.

4.2. *Results on an S-Band Naval Radar*

The pulse length of the radar set was 2 μsec and the receiver constructed for the experiments had a bandwidth of 1 Mc/s. The receiver incorporated the facility that it could be switched at will to be either logarithmic or linear. The receiver was followed by a simple resistance-capacitance differentiating circuit (high-pass filter) to remove the mean value of the clutter and noise fluctuations.

The so-called 'linear' receiver has in fact a linear relationship between input and output signal only over a small range of output voltage. For instance, with a receiver of 10-Mc/s bandwidth, the relationship remains roughly linear for output signals up to about 18 dB above the normal level of output noise, thence upwards to about 30 dB above noise the relationship is roughly logarithmic, after which the receiver saturates, all further amplitude discrimination between signals being lost.

The effect of following the linear receiver by a differentiating circuit is to remove the mean level of the clutter and noise fluctuations at its output, and this in itself has a marked effect in reducing the intensity of the clutter region on the p.p.i. The fact that the linear receiver has itself a limited logarithmic range lends advantage to this process and certain civil marine radars with linear receivers have optional differentiation (by operation of a switch) which goes a considerable way to reducing the intensity of rain-clutter

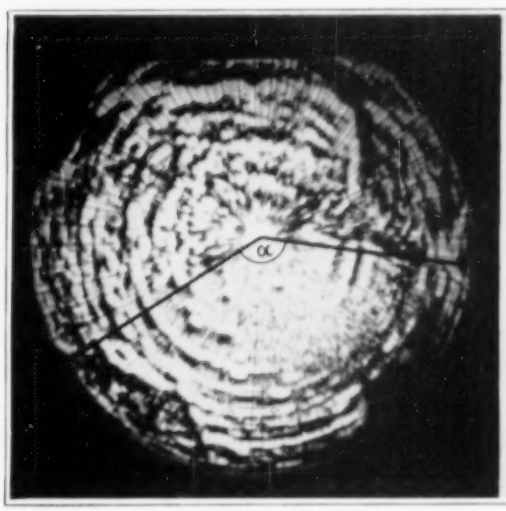


Fig. 8. *Linear receiver followed by differentiating time constant = T (2 μs).*

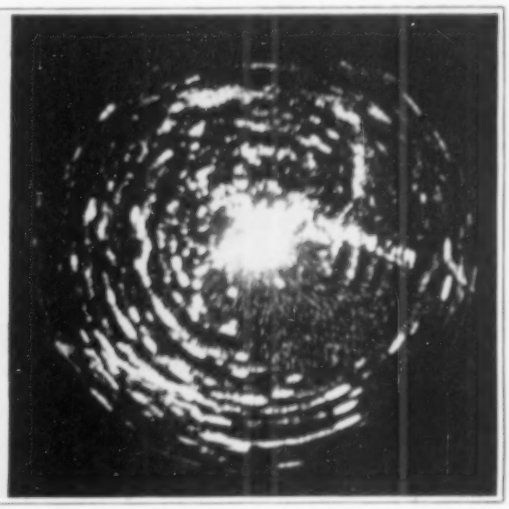


Fig. 9. *Conditions as Fig. 8, but with logarithmic receiver. Photographed ½ min. after Fig. 8.*

paints on the p.p.i., although it cannot eliminate them as is sensibly the case when a logarithmic receiver is followed by differentiation.

The comparison therefore with which this article is concerned, is of the linear receiver plus differentiation and the logarithmic receiver plus differentiation. Admittedly the results from the linear receiver (on sea clutter) could be immediately and immensely improved by the application of swept gain (s.t.c.), but this is a manual control requiring skilled adjustment, and the comparison it is desired to demonstrate is between two automatic devices.

Figs. 8 and 9 show examples of sea clutter photographed on the 12,000 yards range scale of the p.p.i. The sea aspect of the radar set was restricted to the angle α shown in Fig. 8 and it will be noted that clutter extends out to about 9,000 yards. It will be seen that the clutter background is largely eliminated by the combination of logarithmic receiver plus differentiation, but its presence can still be detected as a somewhat coarser type of noise background than is normal. This is mainly because of the trace-to-trace correlation of sea clutter, which as stated in Section 2 may extend over the beam-width of an aerial for a rotating aerial. The effect of this correlation is explained lower down.

Figs. 10 and 11 show photographs of a p.p.i. display with extensive rain clutter. There are heavy rain storms in the areas marked A and B. It will be noted that the logarithmic receiver effectively eliminates the rain clutter, so that those targets which produce echoes greater than that of the volume of rain illuminated at the same time, will paint upon a p.p.i. a background which is indistinguishable from the usual noise background.

Fig. 12 is interesting in that the radar aerial has been stopped, so that the beam is directed over the sea and photographs of an A-scan fed from the linear and logarithmic receivers have been taken. In Fig. 12(a) the first one-third of the trace to the right of the ground wave is occupied by sea clutter which gives place to rain clutter in the middle third, and then to inherent receiver noise. The structure of the rain clutter is not noticeably distinguishable from the inherent noise but the sea clutter has the appearance of a coarser structure due to the trace-to-trace correlation previously mentioned. It is clear from Fig. 12(b) that the r.m.s. amplitudes of the sea clutter, rain clutter, and inherent noise have been equalized by the logarithmic receiver followed by differentiation. The trace-to-trace correlation, however, will still make the sea-clutter region more prominent than noise on the p.p.i., because the clutter echoes tend to paint in one particular spot of the tube face for several successive p.p.i. traces, thus giving more energy to those particular spots than would inherent noise.

Figs. 12(c) and 12(d) are similar photographs to Figs. 12(a) and 12(b) but sea clutter and inherent noise only are present. It is of interest

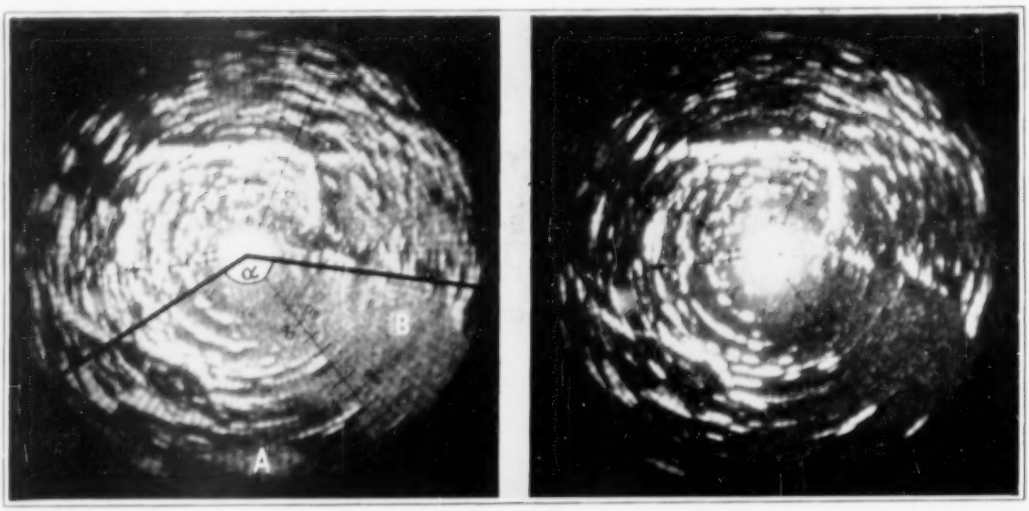


Fig. 10. *Linear receiver, 2 μs time-constant differentiation, sea sector = α. Rain clutter and sea clutter over most of the picture, heavy rain in areas A and B. Range 15,000 yards.*

Fig. 11. *As Fig. 10, but logarithmic receiver in place of linear. Photograph taken ½ minute after Fig. 10. Figs. 10 and 11 taken several days after Figs. 8 and 9.*

to note in the inherent noise region the rather more well-defined level of the noise peaks on the logarithmic receiver due to the compressing action of the logarithmic characteristic.

4.3. *Results on an X-Band Civil Marine Radar*

A logarithmic receiver of similar type was constructed and installed in a Decca Type 12 Civil Marine Radar. The pulse length of the radar set was 0·2 μsec and the bandwidth of the receiver was made 10 Mc/s. This receiver also had the facility of being instantly convertible to a linear receiver by operation of a switch.

Figs. 13 to 16 show two pairs of p.p.i. photographs of sea clutter using the linear receiver followed by differentiation, and the logarithmic receiver followed by differentiation. The captions show the differentiation time constants employed in each case. These photographs were taken from a radar on the coast at Southsea (the lower half of each picture represents the sea aspect) during a defence exercise in which many small craft were involved. It is not often that sea clutter conditions happen to coincide with the presence of numbers of small craft close in shore as occurred in this instance. The range scale in use for these pictures was 3·0 nautical miles, and sea clutter extends to a little beyond one nautical mile. Fig. 16(a) gives a key to some of the echoes in Figs. 15 and 16. Comparison of Figs. 14 and 16 with 13 and 15 shows the great reduction in intensity of sea clutter when the logarithmic receiver is used. The complete elimination of sea clutter, however, which would be expected from the theoretical treatment of Section 3, is not obtained. While this is largely because of the trace-to-trace correlation effect, the elimination of clutter on the X-band radar seemed rather less complete than on the S-band radar, and it looks as if some additional factor is reducing the effectiveness of the logarithmic receiver on the X-band equipment. The anti-clutter operation of the logarithmic receiver is, as shown in Section 3, effective for an amplitude probability distribution of the clutter of Rayleigh form. Now the pulse length and beam-width of the S-band radar were 2 μsec and 5° respectively compared with 0·2 μsec and 1·7° for the X-band radar. The area of sea illuminated by the S-band pulse at a given range is therefore some 30 times that of the X-band pulse at a similar range. If the sea surface is considered as being made up of a number of scatterers, upon which the energy of the radar beam impinges, there will, on an area basis, be roughly 30 times as many scatterers illuminated within a range element of the beam of the S-band radar as are embraced by the equivalent range element of the X-band radar. It was stated in Section 2 that the amplitude probability distribution of sea clutter at any given range 'across' a sequence of radar traces is more or less a Rayleigh distribution, a fact which has been established for low-definition radars of pulse length and beam-width similar to the S-band radar under consideration here. However, in the case of the high-definition X-band radar, where the number of scatterers illuminated by the beam within the pulse length is so much less, it is possible that the amplitude probability distribution may depart significantly from the Rayleigh

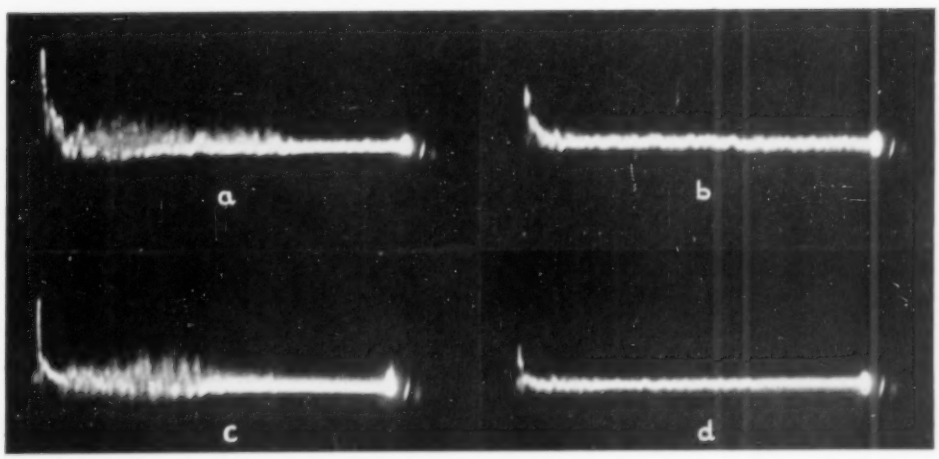


Fig. 12. *Photographs of clutter on 'A' type display; range scale* 15,000 *yards. (a) Linear receiver, differentiating time constant* 2*T, sea clutter extending into rain clutter, and then noise; (b) as (a) but with logarithmic receiver, rain clutter had spread to full range before exposure; (c) linear receiver, differentiating time constant T, sea clutter and noise only; (d) as (c) but with logarithmic receiver in place of linear.*

form. This might reduce the effectiveness of the logarithmic receiver on high-definition radars, but is of course in no way an argument against such radars. For the beam-widths and pulse lengths under consideration here, the signal-to-clutter ratio (for a given radar) is to a first approximation inversely proportional to the product of beam-width and pulse length, so that the advantage in improved signal-to-clutter ratio by going to the higher definition is obvious.

4.4. *Differentiation Losses and Effects*

The purpose of the differentiating circuit is to remove the mean level of the clutter, and it may be assumed that a time constant of several times the pulse length could safely be used to do this. In practice, however, it is as well to restrict the differentiating time constant to about one and a half times the pulse length, since longer time constants result in some extension of the ground-wave region at the centre of the p.p.i. The use

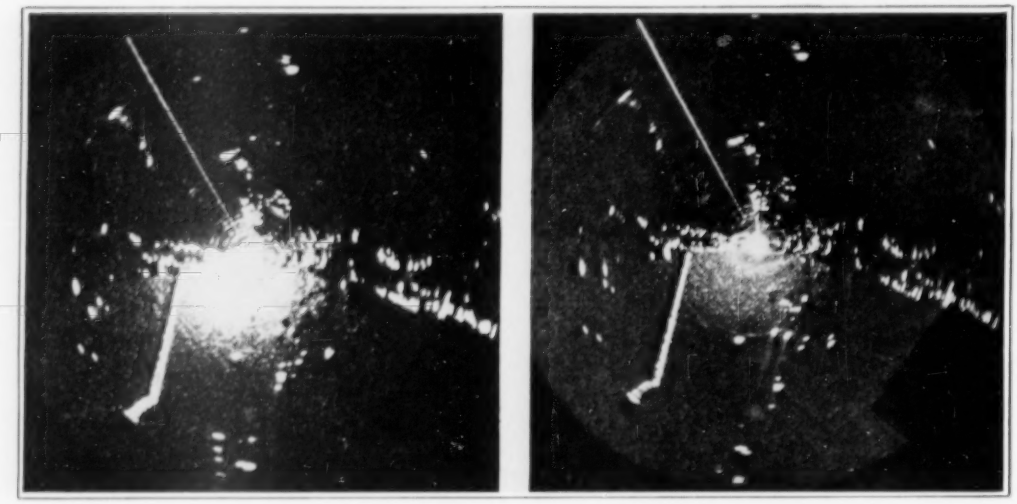


Fig. 13 (left) *for linear receiver and* Fig. 14 (right) *for logarithmic receiver, followed by differentiation; forward and backward time constants* 1·5 *T and* 0·5 *T respectively.*

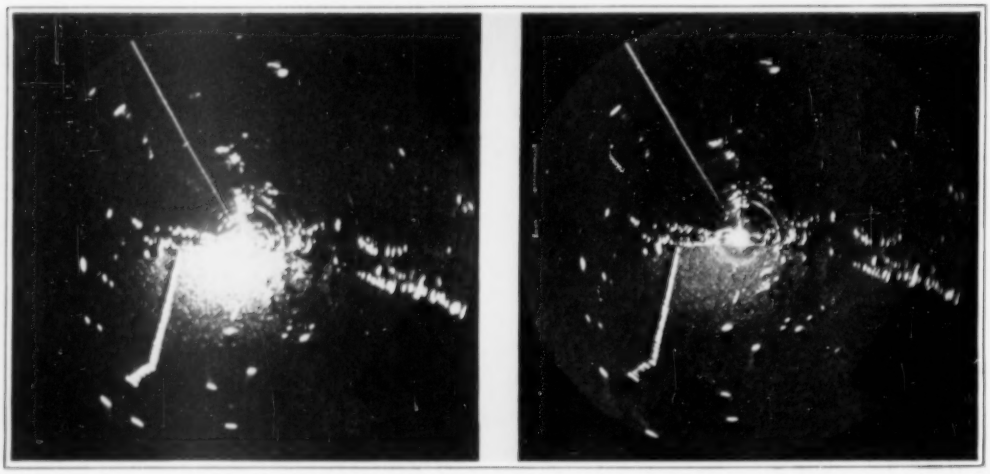


Fig. 15 (left) *for linear receiver and* Fig. 16 (right) *for logarithmic receiver, followed by differentiation; forward and backward time constants* 1·5 *T and* 0·5 *T respectively.*

of a time constant of this order gives rise to shadows immediately after each echo, due to the time taken for the differentiating capacitor to discharge after the echo pulse has ceased. It is usual, therefore, to connect a germanium diode across the differentiating resistance in such a way that it presents a very low resistance to discharge currents from the capacitor, so that discharge takes place much more quickly than charge, the shadows after echoes thereby being rendered negligible in extent. In this manner, recovery times which are a small fraction of the pulse length can be obtained.

When a linear receiver is followed by such an RC differentiating circuit, with a forward time constant of $1{\cdot}5T$ and backward time constant of $0{\cdot}2T$, there appears to be no reduction of signal-to-noise ratio on small signals when compared with the undifferentiated picture. When the same differentiating circuit follows a logarithmic receiver, however, there is a reduction in signal-to-noise ratio of not less than 3 dB, which is of course a considerable price to pay for any advantage offered by the logarithmic receiver.

Fig. 17 (a, b and c) show the photographs of an A-scope trace in which a small echo superimposed upon inherent noise can be seen. The A scope is showing the output of the linear receiver. In Fig. 17(a) the output is not differentiated, but in (b) the output is passed through an RC differentiating network in which the forward time constant is $1{\cdot}5T$ and the backward time constant also $1{\cdot}5T$. In (c) the output is passed through an RC differentiating network in which the forward time constant is $1{\cdot}5T$ and the backward time constant $0{\cdot}2T$. There is no noticeable loss in signal-to-noise ratio between the result shown in the differentiated outputs (b) and (c) and the undifferentiated output (a).

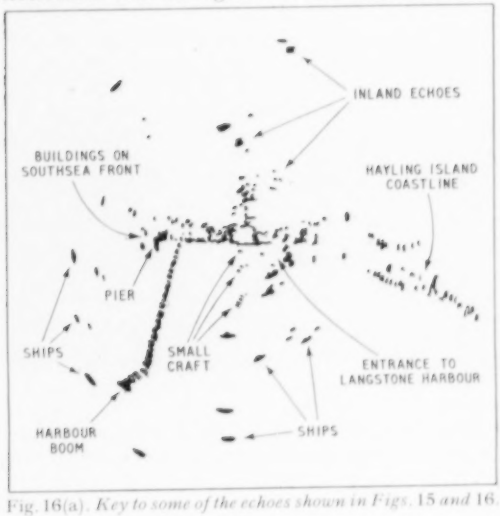


Fig. 16(a). *Key to some of the echoes shown in Figs.* 15 *and* 16.

Fig. 17 (d, e and f) show photographs of the same A-scope trace but fed from the logarithmic

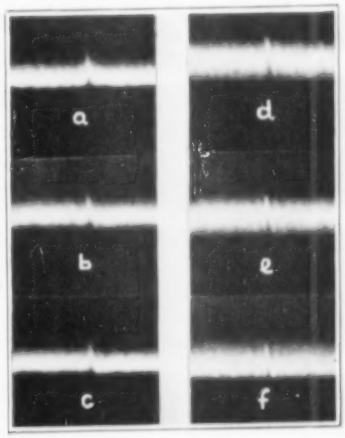


Fig. 17. *Effects of differentiation on weak signals with linear and logarithmic receivers. (a) Linear receiver output, not differentiated; (b) same output as (a) differentiated by forward time constant* $1{\cdot}5T$, *backward time constant* $1{\cdot}5T$; *(c) same output as (a) differentiated by forward time constant* $1{\cdot}5T$, *backward time constant* $0{\cdot}2T$; *(d) as (a) but with logarithmic receiver; (e) as (b) but with logarithmic receiver; (f) as (c) but with logarithmic receiver* (T = *radar pulse length*).

receiver. The sequence is identical with the previous one in respect of the parameters of the differentiating circuit. It will be noticed in (d) that the small echo appears significantly smaller than it does in (a) but this is merely the result of the compressing action of the logarithmic law of the receiver and represents no loss, since the character of the noise is also altered by compression. The signal-to-noise ratio of (d) can therefore be restored to that of (a) by a suitable expansion process (clipping or antilog law) in the video amplifier which follows the logarithmic receiver. Fig. 17(e) does possibly show a small loss in signal-to-noise ratio when compared with the undifferentiated output presented in (d) and, from observations of many p.p.i. pictures, it is thought that a very small loss does occur in differentiation of a logarithmic output even when the backward time constant is kept equal to the forward time constant. It is in the step from (e) to (f), however, where the backward time constant has been reduced to $0{\cdot}2T$, that the real loss occurs which is estimated to be at least 3 dB. Careful observation of (f) will show that the noise has become more feathery than in (d) and (e) and, by holding the edge of a sheet of white paper along the upper level of the noise, first in (d) and (e) and

then in (f), the reduction of the extent by which the small echo protrudes above the noise in (f) as compared with (d) and (e) will be immediately obvious.

This effect was rather baffling until the difference in character between linear and logarithmic noise was considered. Fig. 18(a) represents an attempt to show a twice-noise signal at the output of a linear receiver, presented on an A-scope

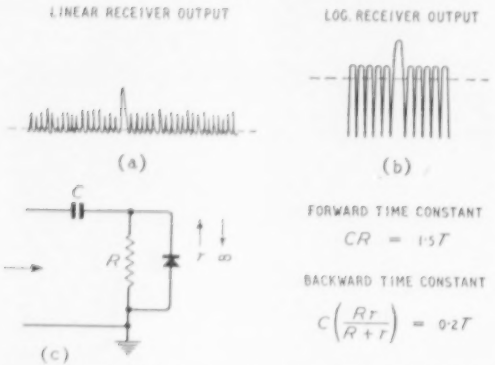


Fig. 18. *Representing small signals at the output of linear and logarithmic receivers and the differentiating circuit to which the outputs are applied.*

trace. Fig. 18(b) shows the same twice-noise signal at the output of a logarithmic receiver; note that the amplitude ratio is no longer 2:1 because of the logarithmic law. To get the same absolute amplitude of signal above noise from the logarithmic receiver as from the linear receiver, much more noise must be accepted from the output of the logarithmic receiver than from the linear receiver. Moreover, whereas the r.m.s. level of the linear noise is near the bottom of the trace, the r.m.s. level of the logarithmic noise is near the top of the trace, the upper excursions of the noise being very much flattened and the lower excursions made very peaky.

In considering the effect of the differentiating circuit of Fig. 18(c) on the outputs of (a) and (b), it should be noted that the negative-going peaks of (a) are small in comparison with the negative-going peaks of (b) and that the differentiating time constant to negative-going peaks is $0{\cdot}2T$ (T = pulse length) while that to positive-going peaks is $1{\cdot}5T$. The positive-going peaks are not significantly differentiated by the long time constant of $1{\cdot}5T$, but the negative-going peaks are differentiated by the much shorter time constant of $0{\cdot}2T$, and the result of this differentiation is to give rise to sharp positive-going peaks which are superimposed on the existing positive-going peaks thus raising the noise level and reducing the signal-to-noise ratio. This effect is negligible on the linear noise of Fig. 18(a) because the noise peaks are only of equal amplitude to the signal but, in Fig. 18(b), where the noise peaks are four times the signal amplitude, the reversed positive-going peaks which result from the short time constant of differentiation ($0{\cdot}2T$) do increase the noise level significantly, and reduce the signal-to-noise ratio by not less than 3 dB. The only way to minimize this effect appears to be to keep the reverse or backward time constant of differentiation equal to the forward time constant, and to accept the shadows which result after the echoes. Where shadows of the order of two pulse lengths cannot be tolerated the reverse time constant may, as a compromise, be reduced to about one-half the pulse length in which case the loss in signal-to-noise ratio is less than 3 dB.

There seems no advantage in using differentiating time constants of less than $1{\cdot}5T$, and with such time constants the author has found no advantage in using the delay-line type of differentiating network as opposed to a simple RC combination.

It is a useful facility to be able to switch the differentiation out of circuit on all ranges of the display, in the absence of clutter, but this is especially desirable on long-range scales of display where the spot size of the c.r.t. is much greater than the pulse length. In this case, echoes on the p.p.i. from coastlines tend to lose considerably in boldness and thickness as a result of differentiation, and it should not be used on long-range scales except in the presence of rain.

4.5. *Restoration of Signal/Noise Ratio in Video Amplifiers*

Considering again the illustrations of Fig. 18(a) and (b) it is seen that, for equal signal amplitudes above noise, the logarithmic receiver will have at its output 'peak' noise amplitudes which are roughly four times as great as the linear receiver. We are here writing in rather loose terms when we talk of 'peak' noise amplitudes since, for the Rayleigh distribution, there is no peak amplitude, all amplitudes up to infinity being present. In reading an A scope, however, the eye does assign a rough peak amplitude to the noise present on the trace and it is to this amplitude we refer.

This disparity in signal-to-noise ratio between the linear and logarithmic outputs is the natural result of using a receiver with a logarithmic law holding uniformly down to noise fluctuations 20 dB below the r.m.s. level. Nothing can be done to remove this disparity *before* the differentiating circuit, for the receiver would then no longer be logarithmic down to 20 dB below r.m.s. noise, and differentiation would not reduce the intensity of the clutter fluctuations to the

constant level of noise. After differentiation, however, it is desirable to clip the base away from the logarithmic noise in the course of video amplification, so as to restore the signal-to-noise ratio of Fig. 18(a). Alternatively, a video amplifier having an anti-log characteristic may be used.

5. Design Notes

5.1. *Noise and Signal Output Amplitudes for Logarithmic Receivers*

For the type of logarithmic amplifier described in reference three, the output voltage increases uniformly with input signal at a rate which is usually not far off 1·0 V per 20 dB. Assuming therefore that the receiver remains logarithmic for signals 80 dB above r.m.s. noise and 20 dB below r.m.s. noise, the maximum signals will have an amplitude of 4 V above r.m.s. noise and the r.m.s. noise level at the output will be something above 1·0 V, usually about 1·3 to 1·5 V. A signal 6 dB above r.m.s. noise will have an amplitude of 0·3 V above noise. A similar signal at the input of a linear receiver would give an amplitude of 0·3 V at the output with 0·3 V of r.m.s. noise present. Assuming, therefore, no loss on differentiation of the logarithmic output, the same video amplifier gain could be used for these two receivers. Base clipping of the logarithmic noise would of course be necessary early in the video amplifier, since something like 40 V of signal is usually necessary to brighten fully an intensity-modulated display, and the video-output valve would have to pass on about 200 V of r.m.s. noise to do this unless clipping were resorted to.

5.2. *Saturation of Final Stages by Noise*

In this type of logarithmic amplifier, the r.m.s. level of noise will usually be saturating the final valve and almost saturating the penultimate valve when the receiver is so working that it is logarithmic to 20 dB below r.m.s. noise level. The valves cannot work at normal voltages in this condition without being seriously overrun, and it is necessary to reduce the screen volts to some safe level (about 140 V) in order to keep the dissipation within normal limits.

To preserve the constancy of the logarithmic law over the whole working range of inputs the fully saturated contributions of each stage to the total output should be equal and the stage gains equal. Alternatively, if the gain of any stage is down by say 50% the saturated contribution of that stage to the output should also be artificially reduced by 50% to compensate.

If all the valves in the amplifier are run at sub-normal screen voltages in order to safeguard the last two valves, the logarithmic law will remain constant over the working range but the total gain of the amplifier will be somewhat reduced by the process, and it may become necessary to add one stage more than would

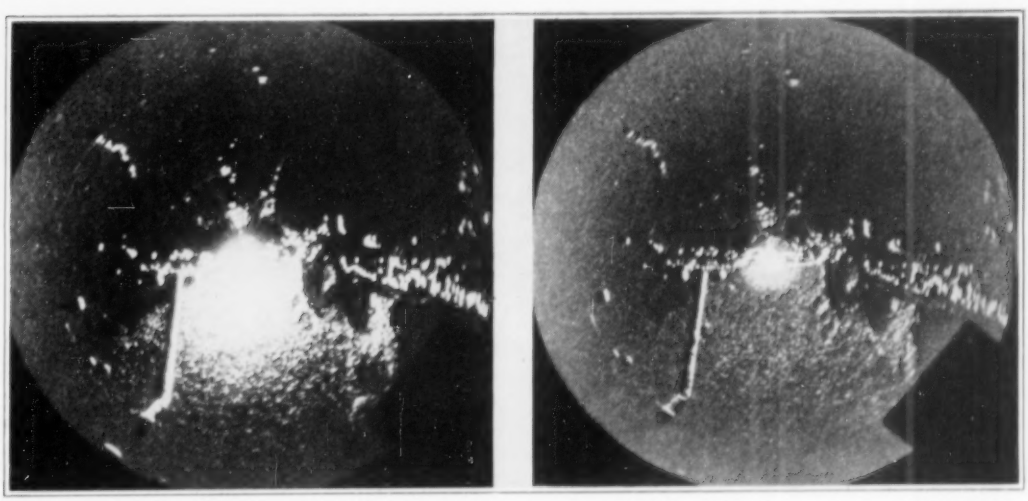


Fig. 19 (left) *for linear and* Fig. 20 (right) *for logarithmic receivers followed by differentiation. Forward time constant = backward time constant = 1·5 T. These pictures were taken with the receiver 'pulsed on' during the brightening pulse of the p.p.i. only. This limits valve dissipation to about one quarter of that of continuous-running conditions and ensures that the last two stages of the logarithmic receiver, which must always be saturated with receiver noise, do not exceed rated dissipation. Targets at 0·5 and 1 mile (7 o'clock) are dan buoys carrying corner reflectors. An X-band civil marine radar (3-miles range scale) was used.*

otherwise be needed. The output of the receiver per decibel of input will also be lowered because the saturated output of each stage will be reduced.

If this condition cannot be accepted the last two valves only may be run at reduced screen voltages; the effect will be to reduce the saturated output of each of these stages, and to reduce their gain very slightly. The reduction of saturated output can be compensated for, and the logarithmic law thus maintained constant, by feeding a suitably larger fraction of the output of these valves into the delay line.

A more elegant solution is to supply the screens with their full rated voltage, but to pulse them on only during the brightening pulse of the cathode-ray tube. Many radar equipments only utilize one-third of the available repetition period even with their longest ranges, and the dissipation can thus be reduced by a factor of three to one without difficulty. A pulse somewhat earlier than the triggering pulse should be used in order to give the screens time to settle down before the incidence of the ground wave. This system works satisfactorily and Figs. 19 to 24 are photographs of the display of a civil marine radar, working from a pulsed receiver under both linear and logarithmic conditions. The performance against sea and rain clutter is indicated in the captions.

5.3. *Tolerances of the Logarithmic Law*

In Section 3 it was shown that, after differentiation, the r.m.s. fluctuations at the output of the logarithmic receiver are given by:—

$$\text{r.m.s. fluctuations} = \frac{a\pi}{\sqrt{24}} \approx 0{\cdot}6a$$

where a is the slope of the logarithmic law $y = a \log_e bx$. If this slope varies over the range, then the r.m.s. fluctuations at the output will not be constant but will depend upon the r.m.s. amplitude of the input signal, since the input signal will decide the operative region of the input–output law of the receiver.

As a p.p.i. display limits at between twice and three times noise, clearly once the display is set up for a given noise level the variations of r.m.s. amplitude of noise and clutter along the trace must not be allowed to vary within anything like a ratio of two to one. In other words the variation of slope of the logarithmic law must be kept well within the limits two to one. In practice, a variation of $\pm$ 20% seems quite acceptable.

6. Trace-to-Trace Correlation

Even with the logarithmic receiver working against an input having a nearly perfect Rayleigh distribution, the trace-to-trace correlation still causes the clutter background to be brighter and coarser than the noise. From time to time suggestions have been put forward for destroying this correlation by sending out successive pulses on different frequencies. Certainly a worth-while improvement in signal-to-clutter ratio could result from destroying the correlation, and the suggestions seem worthy of experiment. A drawback is that either the local oscillator of the

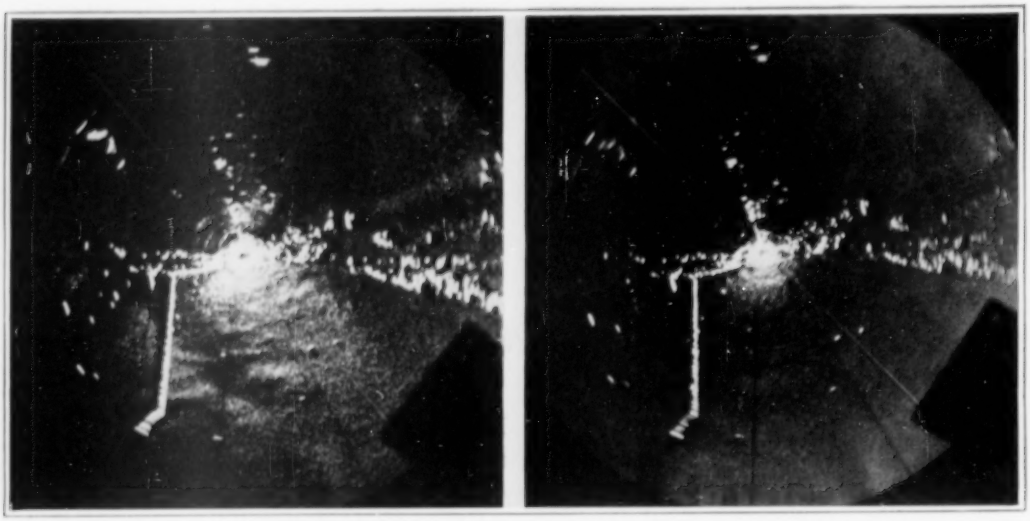
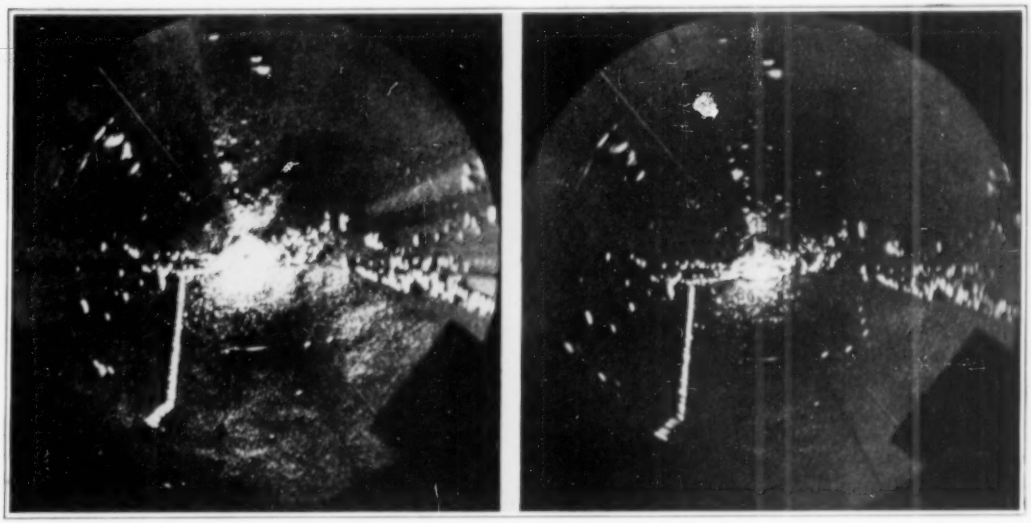


Fig. 21. *A pair of photographs of rain clutter using linear and logarithmic receivers on an X-band civil marine radar. The range scale in use is 3 miles; sea clutter extends to about 0·5 n. miles.* (Left) *For linear receiver and* Fig. 22 (right) *for logarithmic receiver, followed by differentiation. Forward time constant = backward time constant = 1·5 T in each case*

receiver must shift frequency correspondingly with the frequency change of the transmitted pulse or a very wide-band amplifier (100 Mc/s) must be used. Because of its increased noise, such an amplifier would sacrifice most of the improvement obtained, since the signal-to-noise ratio (for weak signals) cannot be adequately restored by restricting the subsequent video bandwidth.



Figs. 23 and 24. *Rain clutter on 3-miles range scale of X-band civil marine radar. The three targets at 4 o'clock (1·5 miles) in Fig. 24 are barely discernible in Fig. 23.* Fig. 23 (left) *for linear receiver and* Fig. 24 (right) *for logarithmic receiver, both followed by differentiation. Forward time constant = backward time constant = 1·5 T in each case.*

7. Conclusions

(1) The logarithmic receiver followed by differentiation will reduce the r.m.s. fluctuations (intensity) of rain clutter and sea clutter more or less to the intensity of the inherent receiver noise, but only provided that it remains logarithmic to 20 dB below r.m.s. level.

(2) With a p.p.i. display the performance against rain clutter is better than against sea clutter because of the pulse-to-pulse correlation of sea clutter.

(3) The performance against sea clutter on a low discrimination radar (5° beam, 2 μsec pulse) is somewhat better than on a high discrimination radar (1·7° beam, 0.2 μsec pulse), possibly because the amplitude probability distribution of the sea clutter on the low-definition equipment conforms more closely to the Rayleigh form than it does with the high-definition equipment.

(4) There is a loss of some 3 dB in signal-to-noise ratio when a receiver logarithmic to 20 dB below r.m.s. noise is followed by a differentiating circuit which uses a short recovery time constant to avoid shadows. For example, a differentiating circuit having a forward time constant of 1·5 times the pulse length and a recovery time constant of 0·2 times the pulse length shows this effect. This loss does *not* occur when the same differentiating circuit follows a linear receiver and is inherent in the lower signal-to-noise amplitude at the output of the logarithmic receiver. The loss can be mainly avoided on the logarithmic receiver by using a backward time constant equal to the forward time constant and tolerating the resulting shadows after echoes.

(5) Differentiation produces considerable display losses on the long-range scale particularly on coastlines. It should only be used on this scale in the presence of rain.

(6) Clipping or an anti-logarithmic characteristic is essential in the video amplifier which follows the differentiating circuit of the logarithmic receiver in order to restore the output signal-to-noise ratio to that of a linear receiver.

(7) For the type of logarithmic receiver considered in this paper, saturation of the final two stages by noise must be accepted if the receiver is to be logarithmic to 20 dB below r.m.s. noise. The dissipation of the valves can be kept within safe limits either by using a sub-normal screen voltage or by using a normal voltage and pulsing the screens on only during the brightening pulse of the p.p.i.

(8) The slope of the input–output law of the logarithmic receiver in volts per decibel should be kept constant within limits of $\pm$ 20%.

(9) A worth-while further improvement in the performance of the logarithmic receiver against sea clutter would result if the pulse-to-pulse correlation could be destroyed by, say, pulse-to-pulse frequency variation.

Acknowledgments

This paper is published with the approval of the Lords Commissioners of the Admiralty, but the responsibility for any statements of fact or opinions expressed rests solely upon the author. The author is indebted to former colleagues in A.S.R.E. for helpful discussions in the course of this work especially to Dr. L. A. G. Dresel, Mr. A. W. Ross, and Mr. W. Struszynski.

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THE IMPEDANCE CONCEPT

Part 1—*Its Relation to Stability and Feedback*

By C. G. Mayo and J. W. Head

(*Research Department, B.B.C. Engineering Division*)

SUMMARY.—Much useful information about the properties of a network can be obtained by means of its impedance or transfer function, which can be determined by repeated application of simple algebraic laws in terms of a variable p closely associated with time differentiation. These laws and operational calculus, by means of which results obtained in the 'p-world' can be transferred to the time world when absolutely necessary, are discussed simply *ab initio* and in the barest possible outline. Stability and damping, however, can be determined by algebraic processes within the 'p-world'; these processes include the determination of partial fractions and the conditions for an algebraic equation to be free from roots with positive real parts, which are discussed in detail starting from first principles. We deduce the conditions for a system to have at least a specified damping rate, and for the stability of a three-stage RC amplifier.

In Part 2 the effect of adding terms in p^4 and p^5 on the roots of a given cubic in p is discussed more fully. It is assumed that the time constants associated with the amplifier are not under the designer's control; the effect of adding a 'step circuit' on the gain and the maximum feedback obtainable are therefore fully considered.

1. Introduction

RECENT developments in the whole field of servo and feedback-amplifier design make it doubtful whether the historical approach to the subject can any longer be regarded as the natural and most useful one. Our present approach is based upon the 'impedance concept' which is now well known. Impedances and the associated transfer functions are expressed, usually algebraically, in terms of a variable $p = \alpha + j\omega$ which is closely associated with time differentiation. Under steady-state conditions, we can regard α as zero, so that p takes the familiar form $j\omega$. We prefer, however, to work throughout in the 'p-world', where much relevant information can be obtained, usually by purely algebraic processes; we seldom have to carry out a 'translation' into the time or frequency worlds. In the 'p-world' we do not have to distinguish between steady-state and transient conditions. The fundamental properties of the 'p-world' and their relevance to practical considerations are discussed in a simple and elementary manner by Carter[1]. Operational calculus is the process by which the frontier between the 'p-world' and the familiar world of time and space is crossed in both directions. A comprehensive 'grammar' and 'dictionary' for 'translation' from 'p-language' to time language and vice versa (without proofs) is provided by McLachlan and Humbert[2]. Certain properties of algebraic equations (in the p-world) are also relevant; many of these are listed in reference 1 (pp. 115–116), for equations of degree not exceeding 4. We have included only sufficient reference, without proof, to these results for clarity. Many of the ideas we here set forth are not new, but their power to clarify fundamental thought about the whole subject of feedback and servo design has perhaps been inadequately exploited. Various difficulties inherent in the historical approach are irrelevant to our present approach.

We include, with proof in the Appendix, a general expression of the condition for a circuit with an algebraic characteristic equation of degree n to be stable. This result is given by Guillemin[3] and is also discussed by Morris and Head[4] from the purely algebraic standpoint. The pioneer in this field, however, was Routh[5]. Frazer and Duncan[6] also obtained, as early as 1928, determinantal conditions of stability which are somewhat unwieldy for a high value of n. We deduce the condition for a circuit to have

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damping at least as rapid as $e^{-\alpha t}$, and apply these results to the case of a three-stage RC amplifier to which we wish to add suitable elements to obtain greater feedback without producing instability and without excessive loss of gain.

2. Fundamental Impedance Laws and Properties of the 'p-World'

We assume the following laws for networks with lumped elements:

(a) The impedances of a resistance R ohms, an inductance L henrys and a capacitance C farads are respectively R, pL and $1/pC$ ohms.

(b) Two impedances Z_1 and Z_2 in series are equivalent to a single impedance $Z_1 + Z_2$.

(c) Two impedances Z_3 and Z_4 in parallel are equivalent to a single impedance $Z_3Z_4/(Z_3 + Z_4)$.

(d) Within the p-world, p may be manipulated algebraically.

By repeated application of (a), (b), (c) and (d) the impedance Z of any network with lumped elements can be found. If the input voltage as a function of time is $V_1(t)$, the input current is $i_1(t)$ and the output voltage is $V_2(t)$, then operational calculus (as we shall see in Table 1) enables us to determine a unique corresponding function of p for each; we shall call these functions respectively V_1, i_1 and V_2. We then have a generalization of Ohm's Law for a network equivalent to a single impedance Z:

(e) $V_1 = i_1 Z$.

Knowing V_1 and Z we can therefore deduce i_1, and hence $i_1(t)$ by operational calculus. We have seen that Z is a function of p associated with the network; V_2/V_1 is another such function, called the transfer function. Our final fundamental law is

(f) If the transfer functions of two networks are T_1 and T_2, the transfer function of the single network formed by connecting these two networks in cascade is $T_1 T_2$, provided that the two networks are separated by a valve so that there is no interaction.

The very brief Table 1 of time functions $V_1(t)$ and the associated 'operationally equivalent' functions of p will be sufficient for our present purpose. In all cases $V_1(t)$ is to be taken as zero when $t < 0$, so that all functions tabulated have discontinuities, or discontinuous derivatives, when $t = 0$.

If $V_1(t)$ corresponds to V_1, then

(g) $\dfrac{d^r V_1(t)}{dt^r}$ corresponds to $p^r V_1 - \sum_{s=0}^{r-1} p^{r-s} \left[\dfrac{d^s V_1(t)}{dt^s}\right]_{t=0}$

(h) $\lim_{t \to 0} V_1(t) = \lim_{p \to \infty} V_1$.

(i) $\lim_{t \to \infty} V_1(t) = \lim_{p \to 0} V_1$.

For a 'dead' circuit the second term in (g) is absent. (h) and (i) frequently give us sufficient information about $V_1(t)$ and its derivatives if we know V_1, so that it is not necessary to determine $V_1(t)$ explicitly by means of Table 1.

3. Application to a Series *LCR* Circuit

Let us apply these assumptions to the series LCR circuit of Fig. 1 considered in detail by Carter[1] (p. 28).

By assumptions (a) and (b), the total impedance is

$$pL + R + \frac{1}{pC} \quad (1)$$

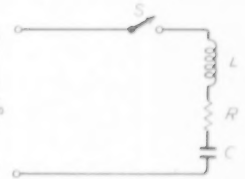


Fig. 1. *A series LCR circuit.*

If therefore the switch S is closed at $t = 0$, the input voltage is by Table 1 operationally equivalent to V_0, and by assumption (e) we have

$$i = V_0 \Big/ \left\{ pL + R + \frac{1}{pC} \right\} \quad \ldots \quad (2)$$

Carter considers the capacitor voltage v instead of i; applying assumptions (a) and (e) to the capacitor only, we have

$$v = \frac{1}{pC} \cdot i$$

Hence eliminating i and using assumption (d)

$$v = \frac{V_0}{LC} \cdot \frac{1}{p^2 + \dfrac{R}{L} p + \dfrac{1}{LC}} \quad \ldots \quad \ldots \quad (3)$$

as found by Carter. Now suppose that the zeros of the denominator in (3) are $-\alpha$ and $-\beta$ so that

TABLE 1*
Time Functions $V_1(t)$ and Operational Equivalents V_1

$V_1(t)$ $[t > 0]$	a	$t^n/n!$	e^{at}	$e^{at} - 1$	$\sin at$	$\cos at$
V_1	a	$1/p^n$	$p/(p-a)$	$a/(p-a)$	$pa/(p^2 + a^2)$	$p^2/(p^2 + a^2)$

$V_1(t)$ $[t > 0]$	$e^{-at} \cos bt$	$e^{-at} \sin bt$	te^{-at}
V_1	$p\,(p+a)/\{(p+a)^2 + b^2\}$	$pb/\{(p+a)^2 + b^2\}$	$p/(p+a)^2$

*We have here used the Heaviside system in preference to the Laplace.

$$\alpha + \beta = R/L; \quad \alpha\beta = 1/LC \qquad (4)$$

Then in order to apply Table 1, we must first express the right-hand side of (3) in partial fractions. When (as usually happens in circuit problems) the numerator of the fraction is of lower degree than the denominator, and the zeros of the denominator are real and distinct, this can be done easily, for

$$v = V_0 \frac{\alpha\beta}{(p+\alpha)(p+\beta)}$$

$$= V_0 \left[\frac{\alpha\beta}{(p+\alpha)(\beta-\alpha)} + \frac{\alpha\beta}{(\alpha-\beta)(p+\beta)} \right] \qquad (5)$$

the rule being that to obtain the partial fraction with denominator $(p+\alpha)$, p is replaced by $-\alpha$ in the numerator and denominator of v everywhere except in the factor $(p+\alpha)$ in the denominator, and similarly for the partial fraction with denominator $(p+\beta)$ we replace p by $-\beta$ everywhere in the numerator and denominator of v except in the factor $p+\beta$.

By Table 1, we then deduce

$$v(t) = V_0 \left\{ 1 - \frac{\alpha e^{-\beta t} - \beta e^{-\alpha t}}{\alpha - \beta} \right\} \qquad (6)$$

If α, β are complex, they will be conjugate so that we can write from (4)

$$\alpha = \gamma + j\omega; \quad \beta = \gamma - j\omega \qquad (7)$$

where $\gamma = R/2L, \quad \omega^2 = (1/LC) - (R^2/4L^2)$ (8)

In this case (6) is still valid, but is more conveniently expressed in terms of γ, ω in the form

$$v(t) = V_0 \{1 - e^{-\gamma t}[\cos \omega t + (\gamma/\omega) \sin \omega t]\} \qquad (9)$$

The borderline case between (6) and (9) occurs when $\alpha = \beta = \gamma = R/2L$ and $\omega = 0$. Now if ωt is sufficiently small, $\cos \omega t$ can be replaced by 1 and $\sin \omega t$ by ωt in (9); this suggests that in the borderline case we should expect to find

$$v(t) = V_0 \{1 - e^{-\gamma t}[1 + \gamma t]\} \qquad (10)$$

and this result is in fact correct[1].

From (8) and (9) we see that the damping rate γ is proportional to R provided that there is oscillation ($R^2 < 4L/C$). If, however, $R^2 > 4L/C$, (6) applies and the effective damping rate is the lesser of α, β, that is

$$\alpha = (R/2L) - \{(R^2/4L^2) - (1/LC)\}^{\frac{1}{2}} \qquad (11)$$

(11) decreases as R increases [above $(4L/C)^{\frac{1}{2}}$]. For large values of R we have

$$\{(R^2/4L^2) - (1/LC)\}^{\frac{1}{2}} = (R/2L)\{1 - (4L/CR^2)\}^{\frac{1}{2}}$$
$$= (R/2L)\{1 - (2L/CR^2) + \ldots\ldots\} \qquad (12)$$

so that α in (11) is nearly equal to $1/RC$. If, therefore, R is gradually increased from zero while L and C remain constant, the damping rate will be proportional to R until $R^2 = 4L/C$ (critical damping), when it will reach its maximum value.* When $R^2 > 4L/C$ the effective damping rate, now given by (11), decreases steadily and tends to zero as R tends to infinity. This apparent anomaly for large values of R is explained by further examination of (6), in which α is now small compared to β. Neglecting α/β, (6) reduces to

$$v(t) \approx V_0\{1 - e^{-\alpha t}\} \approx V_0 \alpha t \text{ for small } \alpha t \qquad (13)$$

so that the smallness of α simply means that $V(t)$ is initially proportional to α and t, and takes a long time to grow to its final value V_0.

We notice that assumptions (g), (h) and (i) give us directly from (3) the initial values of $v(t)$ and its first two derivatives just after the switch is closed, and the final value of $v(t)$; thus $v(0) = 0$, $dv/dt = 0$ and $d^2v/dt^2 = V_0/LC$ when $t = 0$, while the final value of $v(t)$ is V_0 and all its derivatives are zero. Assumption (h) ceases to be meaningful if the numerator of V_1 has higher degree than the denominator.

This indicates that we may be able to obtain adequate information about the circuit of Fig. 1 from (3); the full information contained in equations (6) or (9) may not be required, but the nature of the zeros of the denominator of (3) decides the type of behaviour to be expected. In (6) and (9), $v(t)$ contains one or more exponential terms with a negative exponent, because the denominator of (3) has either negative real zeros [$-\alpha$ and $-(R/L) + \alpha$, where α is given by (11)] or complex zeros with negative real parts [$-\gamma \pm j\omega$ where γ, ω are given by (8)] or equal negative real zeros $-R/2L$. Had the denominator of (3) possessed a zero with a positive real part, the associated circuit would have been unstable, because then (6) or (9) would contain one or more terms which would grow exponentially. In general, a passive network cannot be unstable, so that the existence of a zero with positive real part would indicate the presence of a generator. For a quadratic expression

$$a_0 + a_1 p + a_2 p^2$$

there cannot be zeros with positive real parts if a_0, a_1 and a_2 all have the same sign, but this is not a sufficient restriction on the coefficients of an expression of higher degree. This fact becomes important when we wish to determine the maximum feedback obtainable from a feedback amplifier having several stages or to determine what elements should be added to such an amplifier to increase the feedback obtainable. The fact that (3) has a denominator quadratic in p with positive coefficients is sufficient to show that $v(t)$ either consists of two exponential terms with negative exponents or a damped 'cisoidal' oscillation. 'Cisoidal' is a term used by Campbell

*In general, however, critical damping is not quite the same as maximum damping. See, for example, Carter[1] (p. 70).

and Foster[10] to mean 'of the form $e^{j\omega t}$'; the term is convenient when we are mainly concerned with ω, and are not greatly concerned with distinguishing between the sine-term and the cosine-term. More generally, we are likely to obtain in the 'p-world' an equation similar to (3) but involving a denominator D in p which may be of any degree. All we usually need to know is whether the quantity corresponding to $V(t)$ involves exponentially growing or exponentially decaying terms, or 'cisoidal' oscillations growing exponentially or damped exponentially. This can be determined from the nature of the zeros of D. In the next section, therefore, we consider the conditions for a polynomial in p of the third or higher degree to have no zeros with positive real parts. This does involve what at first sight appears to be pure complex-variable algebra, but it is in fact algebra put to its proper use as a labour-saving device. It enables us to be lazy in an enlightened way, so that we do not have to use Table 1 repeatedly to obtain detailed information which has no practical significance. We also deduce easily the conditions for a polynomial in p to have no zeros whose real parts are greater than a given quantity $-\alpha_0$. Such a polynomial is associated with time functions, 'cisoidal' or exponential, damped at least as rapidly as $e^{-\alpha_0 t}$.

4. Conditions of Stability

We wish to determine the conditions under which the polynomial

$$P_n = a_0 + a_1 p + a_2 p^2 + \ldots + a_n p^n \quad (14)$$

shall have no zeros with positive real parts. If in (14) we replace p by $j\omega$, P_n can be rewritten in the form

$$P_n = (a_0 - a_2\omega^2 + a_4\omega^4 - \ldots) + j\omega(a_1 - a_3\omega^2 + a_5\omega^4 - \ldots) \quad (15)$$

$$= E_n + j\,O_n \text{ for brevity} \quad (16)$$

Now suppose that $\omega_1^2, \omega_2^2 \ldots \omega_s^2$ are the values of ω^2 which make E_n zero while Ω_1^2, $\Omega_2^2 \ldots \Omega_t^2$ are the non-zero values of ω^2 which make O_n zero. Then if n is odd, s and t will both be $\frac{1}{2}(n-1)$, whereas if n is even, $s = \frac{1}{2}n$ and $t = \frac{1}{2}n - 1$. The necessary and sufficient conditions for P_n to be free from zeros with positive real parts are then as follows:

(a) the coefficients of P_n must all have the same sign.

(b) $\omega_1^2, \omega_2^2 \ldots \omega_s^2$ and $\Omega_1^2, \Omega_2^2 \ldots \Omega_t^2$ must all be real and positive; we can therefore regard them as arranged in ascending order.

(c) $0 < \omega_1 < \Omega_1 < \omega_2 < \Omega_2 < \ldots$.

This result is proved in the Appendix for the case $n = 4$, and the way in which the proof can be extended to higher values of n is there indicated. The possibility that the ω and/or Ω terms might not be distinct is also briefly considered, but this case is of mathematical rather than practical importance.

We can also easily obtain the condition that all the zeros shall have negative real parts less than $-\alpha$; if P_n satisfies this condition, the associated circuit will only give rise to oscillations or exponential decay damped at least as rapidly as $e^{-\alpha t}$. To obtain this condition, put $q = p + \alpha$ in (14) and let Q_n be the resulting polynomial in q. Then Q_n must be free from zeros in q with positive real parts, and we can apply the criteria just obtained to Q_n instead of to P_n. If

$$Q_n = b_0 + b_1 q + b_2 q^2 + \ldots + b_n q^n \quad (17)$$

then

$$\left.\begin{aligned}
b_n &= a_n \\
b_{n-1} &= a_{n-1} - \binom{n}{1}\alpha a_n \\
b_{n-2} &= a_{n-2} - \binom{n-1}{1}\alpha a_{n-1} + \binom{n}{2}\alpha^2 a_n \\
b_{n-3} &= a_{n-3} - \binom{n-2}{1}\alpha a_{n-2} + \binom{n-1}{2}\alpha^2 a_{n-1} - \binom{n}{3}\alpha^3 a_n \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
b_{n-r} &= a_{n-r} - \binom{n-r+1}{1}\alpha a_{n-r+1} + \binom{n-r+2}{2}\alpha^2 a_{n-r+2} \ldots + (-1)^r \binom{n}{r}\alpha^r a_n \\
b_0 &= a_0 - a_1\alpha + a_2\alpha^2 + \ldots + (-1)^n \alpha^n a_n
\end{aligned}\right\} \quad \ldots \quad (18)$$

and since α is given in advance, the b terms are easily calculated from (18). The symbol $\binom{m}{k}$ in (18) denotes the binomial coefficient $m!/\{k!\ (n-k)!\}$.

The result we have obtained is a perfectly general condition for a polynomial P_n in p to be free from zeros with positive real parts. This result was obtained in this form by Guillemin[3]. It is equivalent to results first obtained by Routh[5] and

discussed fully by Frazer and Duncan[6]. Carter[1] (p. 116) lists the equivalent results for $n = 2, 3, 4$ where they are expressed simply in terms of the coefficients, but for higher values of n the results cannot be expressed simply and explicitly in terms of the coefficients.

We now apply these results to the case of a three-stage RC feedback amplifier.

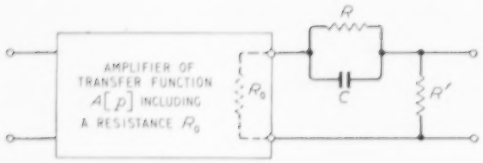


Fig. 2. *Three-stage amplifier with added step-circuit elements to improve feedback.*

5. Application to a Three-Stage *RC* Feedback Amplifier

In the general case of a feedback amplifier, if $A[p]$ is the transfer function of the forward path and $\beta[p]$ is the transfer function of the feedback path, the input voltage v is related to the output voltage V by the equation

$$V/v = A/(1 - A\beta) \quad .. \quad .. \quad .. \quad (19)$$

as first proved by Nyquist[8]. In the case of our three-stage valve amplifier, we shall suppose that

$$A = A_0/(1 + T_1 p)(1 + T_2 p)(1 + T_3 p) \quad (20)$$

where A_0 is a constant and the time constants T_1, T_2 and T_3 are determined by the parameters of the three valves and are not under our control except in so far as some trading of bandwidth for gain is possible. We shall also suppose that the feedback is negative, and that β is a mere negative constant, say $-\lambda/A_0$. The equation determining the stability and damping of the amplifier is then

$$1 - A\beta = 0 \quad .. \quad .. \quad .. \quad (21)$$

This reduces to

$$(1 + \lambda) + (T_1 + T_2 + T_3)p + (T_2T_3 + T_3T_1 + T_1T_2)p^2 + T_1T_2T_3p^3 = 0 \quad .. \quad .. \quad .. \quad (22)$$

In the notation of the last section, the expressions we should have to consider for determining stability would be

$$\left.\begin{aligned} E_3 &= (1 + \lambda) - (T_2T_3 + T_3T_1 + T_1T_2)\omega^2 \\ O_3/\omega &= (T_1 + T_2 + T_3) - T_1T_2T_3\omega^2 \end{aligned}\right\} \quad (23)$$

and for stability the zero of E_3 must be less than the zero of O_3, so that

$$(1 + \lambda)/(T_2T_3 + T_3T_1 + T_1T_2) < (T_1 + T_2 + T_3)/T_1T_2T_3$$

which reduces to

$$\lambda < (T_2/T_3) + (T_3/T_2) + (T_3/T_1) + (T_1/T_3) + (T_1/T_2) + (T_2/T_1) + 2 \quad .. \quad .. \quad (24)$$

a result obtained by one of the authors[9] in 1938. If we now regard λ as a quantity which can be continuously varied from zero upwards, we see that for small values of λ, (22) has three real negative roots in p, and that the effective damping rate is determined by the least of the quantities $1/T_i$; as λ is increased the zero of O_3, which is given by (23), remains fixed, while the zero of E_3 increases steadily with λ. When λ reaches the maximum value determined by (24), the zeros of E_3 and O_3 are equal, so that (22) has a purely imaginary root. It can be shown that there is a unique value λ_0 of λ satisfying (24) such that (22) has equal roots; as λ approaches λ_0 from below the three roots of (22) are real but the two numerically smallest roots approach each other. For λ between λ_0 and the maximum value λ_1 possible without violating (24), (22) has a pair of complex roots with equal negative real parts; these real parts tend to zero as λ approaches λ_1 from below, so that the system then has little damping, and there is a close association between the rate of damping and the difference between the zeros of E_3 and O_3 in (23) even if it is difficult to find an explicit formula expressing this association.

In our second article we determine the effect on the zeros of a cubic P_3 in p if terms in p^4 and p^5 are added. A possible method of obtaining increased feedback is to add a 'step circuit' as in Fig. 2. When this is done, we have three parameters R, C and R' which are under our control, whereas T_1, T_2, T_3, and R_0 are not; R_0 is taken into account in determining T_1, T_2 and T_3 and is merely a resistance which already exists as part of the original amplifier to which it happens to be convenient to attach the step circuit. We therefore consider how the choice of values for R, C and R' (or rather for certain parameters determined by them) affects the degree of feedback obtainable and the overall gain.

APPENDIX

Conditions for a Polynomial to be Free from Zeros with Positive Real Parts

In Section 4 we stated the necessary and sufficient conditions for the polynomial P_n defined by (14) to be free from zeros with positive real parts. We now prove these results for the case $n = 4$ and indicate how the proof may be extended to higher values of n. We also discuss briefly the possibility that the quantities called ω_s and/or Ω_s in Section 4 might not be distinct. Consider, therefore, the polynomial

$$P_4 = a_0 + a_1p + a_2p^2 + a_3p^3 + a_4p^4 \quad .. \quad (A1)$$

and let

$$p = \alpha + j\omega = re^{j\theta} \quad .. \quad .. \quad . \quad (A2)$$

$$\begin{aligned} P_4 &= Re^{j\phi} = (p - p_1)\,(p - p_2)\,(p - p_3)\,(p - p_4) \\ &= R_1e^{j\phi_1} \cdot R_2e^{j\phi_2} \cdot R_3e^{j\phi_3} \cdot R_4e^{j\phi_4} \quad .. \end{aligned} \quad (A3)$$

where $p_k = \alpha_k + j\omega_k$ are the zeros of P_4, so that

$$r = (\alpha^2 + \omega^2)^{1/2}; \quad \theta = \tan^{-1}(\omega/\alpha) \quad .. \quad .. \quad (A4)$$

$$R_k = \{(\alpha - \alpha_k)^2 + (\omega - \omega_k)^2\}^{1/2}; \quad \ldots \quad \text{(A4)}$$

$$\phi_k = \tan^{-1}\{(\omega - \omega_k)/(\alpha - \alpha_k)\}\ (k = 1, 2, 3, 4) \quad \text{(A5)}$$

$$R = R_1 R_2 R_3 R_4;\ \phi = \phi_1 + \phi_2 + \phi_3 + \phi_4 \quad \ldots \quad \text{(A6)}$$

It is shown in reference 7 that if p is made to follow out any closed curve C, the consequent change in ϕ, the argument of P_4 [see (A3)], will be 2π times the number of zeros of P_4 contained within C. We shall give the argument fully, following reference 7, for the case when C is a quadrant of a circle of large radius r_0 bounded radially by the real and imaginary axes as in Fig. 3. If r_0 is sufficiently large, any zero of P_4 which has positive real and imaginary parts can be regarded as within C, and we can also regard the term $a_4 p^4$ as the only important term of P_4 when $|p| = r_0$. As complex zeros of P_4 occur in conjugate pairs, any zero of P_4 in the first quadrant will be associated with a corresponding zero in the fourth quadrant which will also have a positive real part. If therefore the argument ϕ of P_4 undergoes no change when p describes the large quadrant C already specified, P_4 will be free from zeros with positive real parts as required, and if the argument ϕ of P_4 does change, it will change by $2N\pi$ where N is the number of zeros of P_4 with positive real and imaginary parts; P_4 will in this case also have N roots with positive real and negative imaginary part. If p is varied from zero following the closed path shown by arrows in Fig. 3, and r_0 is large, θ remains zero until $p = r_0$, when $\alpha = r_0$ and $\omega = 0$. Round the circular part of the path, r_0 remains constant while θ increases from 0 to $\pi/2$; round this part of the path α is $r_0 \cos\theta$, which decreases steadily, while ω is $r_0 \sin\theta$, which increases steadily, so that $\alpha^2 + \omega^2$ remains constant and equal to r_0^2. When $\theta = \pi/2$, α is zero and $\omega = r_0$. Thereafter, θ remains constant and equal to $\pi/2$, and $p = j\omega$; p thus falls from jr_0 to zero completing the contour.

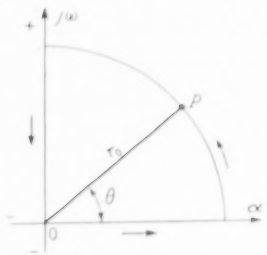


Fig. 3. *A closed contour for p.*

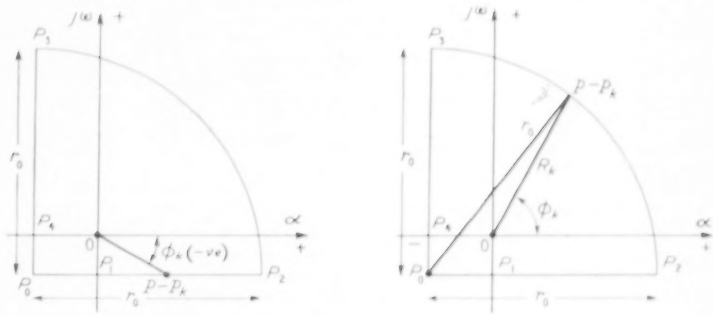
Now consider the behaviour of

$$p - p_k = R_k e^{j\phi_k}$$

as p describes the contour in Fig. 3. R_k and ϕ_k are given by (18), and we shall first suppose that α_k and ω_k are positive, so that the point $p = p_k$ lies within the contour of Fig. 3. The initial position of $p - p_k$ will be the point P_0 $(-\alpha_k, -\omega_k)$ which is in the third quadrant as in Fig. 4(a), and has an argument between $-\pi$ and $-\pi/2$. The locus of the point $(p - p_k)$ will be a quadrant, like that of Fig. 3, but having P_0 as centre instead of the origin.

Initially the argument of $(p - p_k)$ will be that of P_0 which we have taken as between $-\pi$ and $-\pi/2$. As p moves along the real α-axis as in Fig. 4(a), $(p - p_k)$ will move parallel to the real α-axis, and ϕ_k will become $-\pi/2$ at P_1 when $p = \alpha_k$ and ϕ_k will be small and negative when $p = r_0$ at P_2. While p describes the circular part of its contour, Fig. 4(b), since r_0 is large, p and $(p - p_k)$ will differ little, so ϕ_k will increase from a small and negative value to a value just over $\pi/2$ when p reaches the value jr_0 at P_3. As p descends the imaginary axis to the value $j\omega_k$ at P_4, the argument of $(p - p_k)$ increases steadily to π. Finally, when p completes the contour, the argument of $(p - p_k)$ has a value between π and $(3\pi/2)$ which is 2π more than the initial value. If now we suppose either of α_k and ω_k to be negative (or both as in Fig. 5), so that the point $p = p_k$ is no longer within the contour of Fig. 3, the point P_0 is in one of the other quadrants, and the quadrant $P_0 P_2 P_3$ will be found no longer to include the origin. A repetition of the discussion of the changes in ϕ_k shows that ϕ_k now returns to its initial value after p has described the contour of Fig. 3.

It follows that when p describes the contour of Fig. 3, each of ϕ_1, ϕ_2, ϕ_3 and ϕ_4 as defined by (A3) increases by either 0 or 2π, and therefore that the sum ϕ of these angles increases by 2π times the number of zeros of P_4 enclosed within the path of Fig. 3; for sufficiently large r_0, this means 2π times the number of zeros of P_4 lying in the first quadrant.



(a) (b)

Fig. 4(a). *Behaviour of $(p - p_k)$ when p moves from the origin along the positive real axis, and p_k has positive real and imaginary parts.* (b) *Behaviour of $(p - p_k)$ when p describes the circular part of the contour of Fig. 2 and p_k has positive real and imaginary parts.*

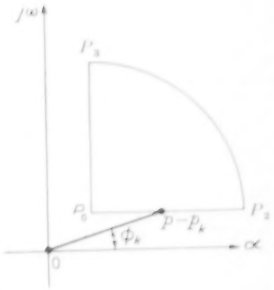


Fig. 5. *Behaviour of $(p - p_k)$ when p describes the contour of Fig. 3 and p_k has negative real and imaginary parts.*

Now consider the behaviour of the polynomial P_4 itself [see equation (A1)]. Initially the argument ϕ of P_4 is zero. We are interested only in the case where all the a terms are positive as it can easily be proved that P_4 must have positive real parts if one of the a terms (other than a_0, a_4) is zero or if they do not all have the same sign. It follows that ϕ remains zero as p moves along the real axis to $p = r_0$. While p describes the circular part of its contour, the predominant term of

the polynomial P_4 is $a_4 p^4 = a_4 r_0^4 e^{4j\theta}$ if r_0 is sufficiently large. Hence the argument ϕ of P_4 is approximately 4θ and increases from 0 to (approximately) 2π. p has now reached the value jr_0, so that ω has the large value r_0, and we have to consider what happens when p descends the imaginary axis and ω decreases to zero. At this stage $p = j\omega$ and P_4 reduces to

$$P_4 = (a_0 - a_2\omega^2 + a_4\omega^4) + j\omega(a_1 - a_3\omega^2) \quad .. \quad (A7)$$

$$= E_4 + jO_4 \text{ for brevity} \quad .. \quad .. \quad .. \quad (A8)$$

E_4 is formed from the terms of even degree in P_4, while O_4 is formed from those of odd degree. We first consider the case $a_2^2 < 4a_0a_4$, so that E_4 is positive for all values of ω. O_4 is negative for $\omega > (a_1/a_3)^{1/2}$ but positive for $\omega < (a_1/a_3)^{1/2}$. When ω has the large value r_0, O_4 is thus large and negative while E_4 is large and positive. The argument ϕ of P_4 is now given by

$$\tan\phi = \frac{O_4}{E_4} = \frac{\omega(a_1 - a_3\omega^2)}{a_0 - a_2\omega^2 + a_4\omega^4} \quad (A9)$$

We have already seen that ϕ is approximately 2π when $\omega = r_0$; (A9) shows that $\tan\phi$ then has the small negative value $-a_3/(a_4 r_0)$, so that ϕ is in fact slightly less than 2π.

From (A9) we see that ϕ cannot pass through an odd multiple of $\pi/2$ unless E_4 is zero and it cannot pass through an even multiple of $\pi/2$ unless O_4 is zero. The detailed implications of this for the case under consideration, when E_4 is always positive and ω steadily decreases, are as follows. E_4 remains positive and O_4 negative until ω reaches the value $(a_1/a_3)^{1/2}$; ϕ therefore remains between 2π and $3\pi/2$ returning to the value 2π when $\omega = (a_1/a_3)^{1/2}$. When $\omega < (a_1/a_3)^{1/2}$, E_4 and O_4 are both positive so that ϕ is between 2π and $5\pi/2$; for small values of ω, however, O_4 is small whereas E_4 is approximately a_0, so the final value of ϕ is 2π, and P_4 has one complex root with positive real part in the first quadrant, and therefore two complex roots with positive real parts altogether. The general behaviour of ϕ as p descends the imaginary axis is sketched roughly in Fig. 6.

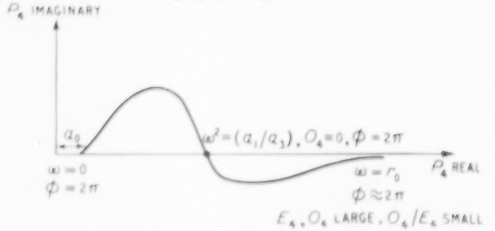


Fig. 6. *Rough sketch of behaviour of P_4 when p descends the imaginary axis and E_4 has no real zeros.*

Now suppose that $a_2^2 > 4a_0a_4$, so that there are two real positive values of ω for which E_4 is zero. If these values of ω and ω_1 and ω_2 $(\omega_2 > \omega_1 > 0)$, E_4 is positive for $\omega > \omega_2$, negative for $\omega_2 > \omega > \omega_1$ and positive for $\omega < \omega_1$, but ω_1 and ω_2 may have any relation to $(a_1/a_3)^{1/2}$. If $\omega_2 > (a_1/a_3)^{1/2} > \omega_1$, we have, as before, that the argument of ϕ of P_4 is slightly less than 2π when $\omega = r_0$, but now it descends to $3\pi/2$ when $\omega = \omega_2$ since for $\omega = \omega_2$, E_4 is zero while O_4 is negative. For $\omega_2 > \omega > (a_1/a_3)^{1/2}$ E_4 and O_4 are both negative, so that ϕ is between $3\pi/2$ and π, reaching the latter value when $\omega = (a_1/a_3)^{1/2}$. Similarly, ϕ is between π and $\pi/2$ for $(a_1/a_3)^{1/2} > \omega > \omega_1$, and ϕ becomes $\pi/2$ when $\omega = \omega_1$. Lastly, for $\omega < \omega_1$, ϕ lies between $\pi/2$ and 0 and ϕ clearly tends to zero with ω. This is sketched roughly in Fig. 7.

In this case, therefore, P_4 has no complex root in the first quadrant; it must therefore have no complex root in the fourth quadrant. If the a terms are all positive, P_4 clearly has no real positive zero, so that P_4 must be entirely free from roots with positive real parts.

It remains to consider the cases in which E_4 is zero for two real positive values ω_1, ω_2 of ω $(\omega_2 > \omega_1 > 0)$ but either $\omega_2 > \omega_1 > (a_1/a_3)^{1/2}$ or $(a_1/a_3)^{1/2} > \omega_2 > \omega_1$. By a similar argument to the foregoing we can show that P_4 must have at least one zero with positive real and positive imaginary part in either of these cases, and therefore at least one pair of conjugate complex roots with positive real parts.

In the general case the only differences are (i) the argument ϕ of P_n will increase by $\frac{1}{2}n\pi$ instead of by 2π as p describes the circular part of its path in Fig. 2,

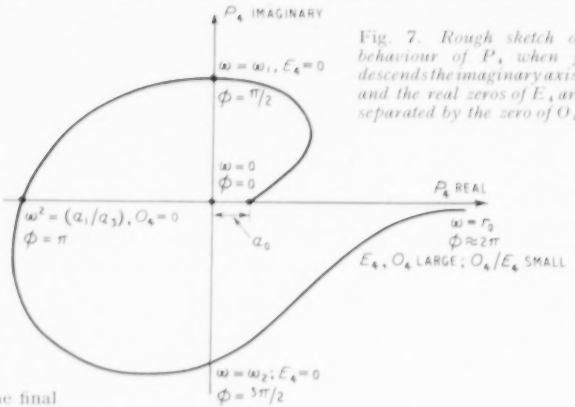


Fig. 7. *Rough sketch of behaviour of P_4 when p descends the imaginary axis, and the real zeros of E_4 are separated by the zero of O_4.*

(ii) E_n and O_n will have a number of roots, and there will therefore be more possible cases to consider. The result, however, is quite clear and definite, and involves the three conditions (a), (b) and (c) already given in Section 4.

We have disregarded the possibility that E_n or O_n in (16) might have equal zeros, for clearly the condition of separation (c) in Section 4 is then violated unless a repeated zero of E_n is also a zero of O_n or vice versa. In this case P_n has a purely imaginary zero, whereas the practical requirement is that all the zeros of P_n should have negative real parts.

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(To be concluded)

CORRESPONDENCE

Letters to the Editor on technical subjects are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

Non-Linearity Distortion

SIR,—I was particularly interested in your January 1956 Editorial comments on non-linearity and its cancellation.

There is one point in connection with even-order harmonic distortion, however, namely, that the assymetry of the output wave produces a d.c. component. This is not transmitted by normal types of interstage coupling, so that a balance arrived at under steady-state conditions would not be maintained accurately under normal programme conditions, where the syllabic variation of the d.c. component would be passed (with a certain amount of attenuation and phase-shift). This in turn would cause a variation in the mean bias of the following stage.

Distortion cancellation is quite a practical proposition in a.m. radio transmitters employing low-level modulation. Where the r.f. driver is a screen-modulated pentode it is possible, by suitable adjustment of the grid drive, to produce a modulation characteristic which is the exact inverse of the linearity curve of the class B final stage, thereby reducing the distortion to a negligible amount.

The adjustment is easily and quickly made using an oscilloscope, and the balance remains exact under all conditions of syllabic variation of the modulation level.

Wanganin, D. C. SUTHERLAND
New Zealand.
6th March 1956.

[In our January Editorial, we assumed a valve with a characteristic of

$$i_a = 0{\cdot}01 + 0{\cdot}0016\ V + 0{\cdot}00006\ V^2$$

and we let $V = -5 + 4 \sin \omega t$, giving

$$i_a = 0{\cdot}00398 + 0{\cdot}004 \sin \omega t - 0{\cdot}00048 \cos 2\omega t$$

We assumed that the a.c. component only was handed on to an identical stage by a coupling with a resistive impedance of 1 kΩ, so that for the second stage

$$V = -5 - 4 \sin \omega t + 0{\cdot}48 \cos 2\omega t$$

and we found that this gave for the anode current of the second stage

$$i_a = 0{\cdot}003987 - 0{\cdot}004115 \sin \omega t - 0{\cdot}0001152 \sin 3\omega t + 0{\cdot}000006912 \cos 4\omega t$$

Now if, in the first stage, the input were a sine wave of only 2-volts peak then, for this stage, $V = -5 + 2 \sin \omega t$ and the anode current is

$$i_a = 0{\cdot}00362 + 0{\cdot}002 \sin \omega t - 0{\cdot}00012 \cos 2\omega t$$

If now the input is suddenly changed from 2 volts to 4 volts, there is a sudden change of the mean anode current from 0·00362 A to 0·00398 A, which is 0·00036 A. This is handed on to the next stage as a change of mean grid voltage of 0·36 volt.

Immediately after the change, therefore, the second stage has

$$V = -5{\cdot}36 - 4 \sin \omega t + 0{\cdot}48 \cos 2\omega t$$

Inserting this in the equation for anode current gives

$$i_a = 0{\cdot}002674688 - 0{\cdot}003712 \sin \omega t - 0{\cdot}000020736 \cos 2\omega t - 0{\cdot}0001152 \sin 3\omega t + 0{\cdot}000006912 \cos 4\omega t$$

Comparing the two expressions, the amplitudes of the third and fourth harmonics are unchanged. The second harmonic is not zero and the fundamental is smaller. Instead of 2·8% third harmonic and 0·168% fourth, we have 0·56% second, 3·1% third and 0·186% fourth harmonic.

The distortion cancellation thus holds fully only under steady-state conditions and is momentarily upset during the transient which occurs on a change of amplitude. A d.c. coupling under the assumed conditions would give even worse results, but that is only because the conditions are not the right ones for d.c. coupling. The ideal is obviously a d.c. coupling and valve characteristics so chosen that, in combination, the overall static characteristic is linear. Nevertheless, even with a.c. coupling, the distortion under transient conditions, although greater than in the steady state, is still less than if there were no cancellation at all. W.T.C.]

Practical Transistor Circuits

SIR,—Some of the most interesting engineering possibilities of the transistor are associated with its small physical size: is it right then to conclude, as in your February Editorial, that a practical value for the circuit constant C_s is 35 μF?

This represents a component physically many times larger in volume than the small-sized transistors which are now or will presently be available. Similarly, one finds that practical designs for output transformers associated with transistors, having collector dissipation in the watt range, are considerably bigger than the transistors themselves.

The optimum design of a transistorized circuit will be indicated by the answers to these queries:

(a) Is it physically possible to produce 'large' electrolytic or tantalum capacitors of the order of 35 μF, for a few volts working level, in the lineal size range of the smallest transistors?

(b) Can we look forward to smaller transformers through the more effective use of materials of higher flux density and higher permeability than are now available commercially?

(c) Alternatively, shall we accept less gain per transistor stage, using a larger number of transistors, in order to have smaller magnitudes for the values of the coupling components?

Future practice may be represented by (c), just as with small pentode valves we are usually prepared to accept a stage amplification of × 100, although, as is well known, × 1000 or nearly the full valve amplification factor itself can be realized, at the expense of an inconveniently large anode load and high supply voltage.

Fortiphone Ltd., D. L. JOHNSTON
London, W.1.
16th February 1956.

AUTOMATION

The Proceedings of the Radio-Electronics-Television Manufacturers' Association Symposium on Automation is a record of the papers presented at the symposium held in the University of Pennsylvania, Philadelphia, Pennsylvania, U.S.A., on 26th and 27th September 1955. Copies cost $5 from Engineering Publishers, G.P.O. Box 1151, New York 1, N.Y., U.S.A.

NEW BOOKS

Services' Textbook of Radio; Vol. 3, Electronics

By J. THOMSON, M.A., D.Sc., F.Inst.P., M.I.E.E. Edited by the Technical Staff of *Wireless World*. Pp. 259 + ix. H.M. Stationery Office, York House, Kingsway, London, W.C.2. Price 12s. 6d.

This book is one of a series of seven volumes which will cover the whole field of radio and radar. Although the third volume of the series, it is the first to be published.

"Electronics" deals mainly with the thermionic valve, but c.r. tubes, photo-electric devices and semiconductor 'valves' are treated also. Certain sections are marked for omission on a first reading and the level of treatment is then suited to the beginner; when read to include these marked parts, and still more with the appendixes, the treatment reaches a fairly advanced level.

***Wireless & Electrical Trader* Year Book 1956** (27th Edn)

Pp. 344. Trader Publishing Co. Ltd., Dorset House, Stamford Street, London, S.E.1. Price 12s. 6d.

Hi-Fi Loudspeakers and Enclosures

By ABRAHAM B. COHEN. Pp. 360 + vii. John F. Rider Publisher Inc., 480 Canal Street, New York 13, N.Y., U.S.A. Price $4.60.

Introduction to Color TV (2nd Edn)

By M. KAUFMAN and H. THOMAS. Pp. 156 + iv. John F. Rider Publisher Inc., 480 Canal Street, New York 13, N.Y., U.S.A. Price $2.70.

Multivibrators

Edited by ALEXANDER SCHURE, Ph.D., Ed.D. Pp. 48 + iv. John F. Rider Publisher Inc., 480 Canal Street, New York 13, N.Y., U.S.A. Price 90 cents.

Color Television Standards

Edited by DONALD G. FINK. Pp. 520 + xii. McGraw-Hill Publishing Co. Ltd., 95 Farringdon Street, London, E.C.4. Price 64s.

This book comprises papers selected from the Proceedings of the Second National Television System Committee, whose deliberations from January 1950 to July 1953 resulted in the present American colour television standards.

Electronics

By A. W. KEEN, M.I.R.E., A.M.I.E.E. Pp. 256. Ward Lock & Co. Ltd., 143 Piccadilly, London, W.1. Price 25s.

This is a semi-popular book for readers with "a sound knowledge of elementary electricity and magnetism". In spite of its title, it covers almost everything in which radio valves are employed, including radar.

High Vacuum Technique (3rd Edition, revised)

By J. YARWOOD, M.Sc., F.Inst.P. Pp. 208 + viii. Chapman & Hall Ltd., 37 Essex Street, London, W.C.2. Price 25s.

PREMIUMS FOR TECHNICAL WRITING

The Radio Industry Council has awarded six premiums of 25 guineas each for articles published in the public technical press during 1955. They are to:—

A. W. M. Coombs, for his article "'Memory' Systems in Electronic Computers", which appeared in *Communications and Electronics* for March 1955.

R. A. Bracewell, for his article "An Infra-Red Radiation Pyrometer", which appeared in *Electronic Engineering* for June 1955.

Alan E. Crawford, for his article "Progress in High Power Ultrasonics", which appeared in *British Communications and Electronics* for August and September 1955.

A. H. Beck, T. M. Jackson and J. Lytollis, for their article "A Novel Gas-Gap Speech Switching Valve", which appeared in *Electronic Engineering* for January 1955.

The remaining two premiums have been awarded jointly to the authors of three articles which appeared in the *Post Office Electrical Engineers' Journal* for July and October 1955.

I. A. Ravenscroft and R. W. White, "A Frequency Modulator for Broad-Band Radio Relay Systems".

R. W. White and J. S. Whyte, "Equipment for Measurement of Inter-Channel Crosstalk and Noise on Broad-Band Multi-Channel Telephone Systems".

J. S. Whyte, "An Instrument for the Measurement and Display of V.H.F. Network Characteristics".

The awards were made at a luncheon held by the Public Relations Committee of the Radio Industry Council on 8th March 1956.

STANDARD-FREQUENCY TRANSMISSIONS

(Communication from the National Physical Laboratory)

Values for February 1956

Date 1956 February	Frequency deviation from nominal: parts in 10^8	
	MSF 60 kc/s 1429-1530 G.M.T.	Droitwich 200 kc/s 1030 G.M.T.
1	−0·1	+1
2	−0·2	0
3	−0·2	0
4	−0·1	0
5	−0·1	+2
6	−0·1	+2
7	−0·1	+1
8	−0·1	+1
9	−0·1	+2
10	−0·2	+2
11	−0·2	+4
12	−0·2	+2
13	−0·2	+2
14	−0·1	+2
15	−0·2	+2
16	−0·2	+2
17	−0·1	+2
18	−0·1	+3
19	−0·1	+4
20	−0·1	+4
21	−0·1	+5
22	−0·1	0
23	0·0	0
24	−0·1	0
25	−0·1	0
26	−0·1	−2
27	−0·1	−2
28	−0·1	−2
29	−0·2	−2

The values are based on astronomical data available on 1st March 1956.

DATA for the Designers File

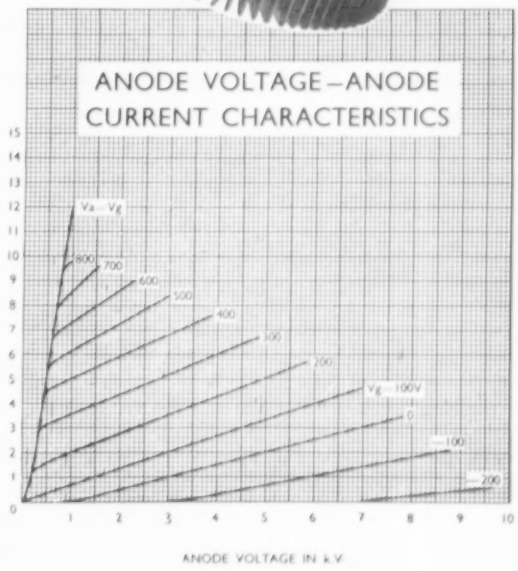
The Ediswan ESA.1500 is of particular interest to Designers of Industrial R.F. heating equipment.

This modern valve, the Ediswan ESA.1500, is a directly heated, forced air cooled, thoriated filament triode primarily intended for operation as an Oscillator in R.F. heating equipment. It may also be used as an R.F. or A.F. power amplifier.

RATING

Filament Voltage (volts)	V_f	8
Filament Current (amps)	I_f	26
Maximum Anode Voltage (volts)	V_a (max)	5,500
Maximum Usable Filament Emission (amps)	F_{em}	6
Maximum Anode Dissipation (kW)	W_a	2.5
Mutual Conductance (mA/V)	g_m	*7.5
Amplification Factor	μ	*24
Anode Impedence (ohms)	r_a	*3,200
Maximum Operating Frequency at full rating (Mc/s)		40
R.F. Power Output (kW)		6

* Taken at $V_a = 5{,}000$v; $I_a = 400$ mA.



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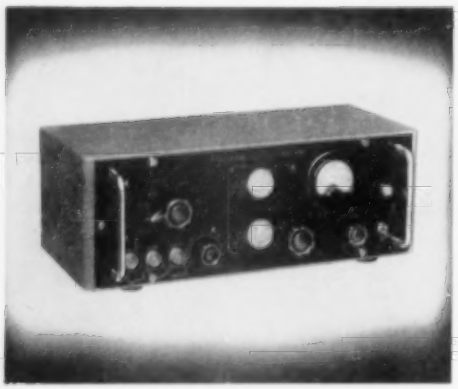

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Alternatively, the small signal voltage can be fed to a straightforward d.c. valve amplifier, while a fraction of the voltage is taken to an a.c. amplifier using an " each-side-stable " Carpenter relay. One side-contact of the relay is used alternately to " earth " and to " free " this input voltage, thereby converting it to square-wave a.c., while the other side-contact demodulates the amplified a.c. output. This output is then fed back to the control grid of the original d.c. amplifier, thereby eliminating any tendency of the output to drift.

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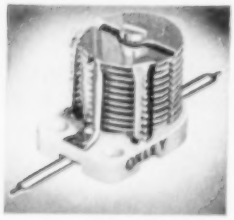

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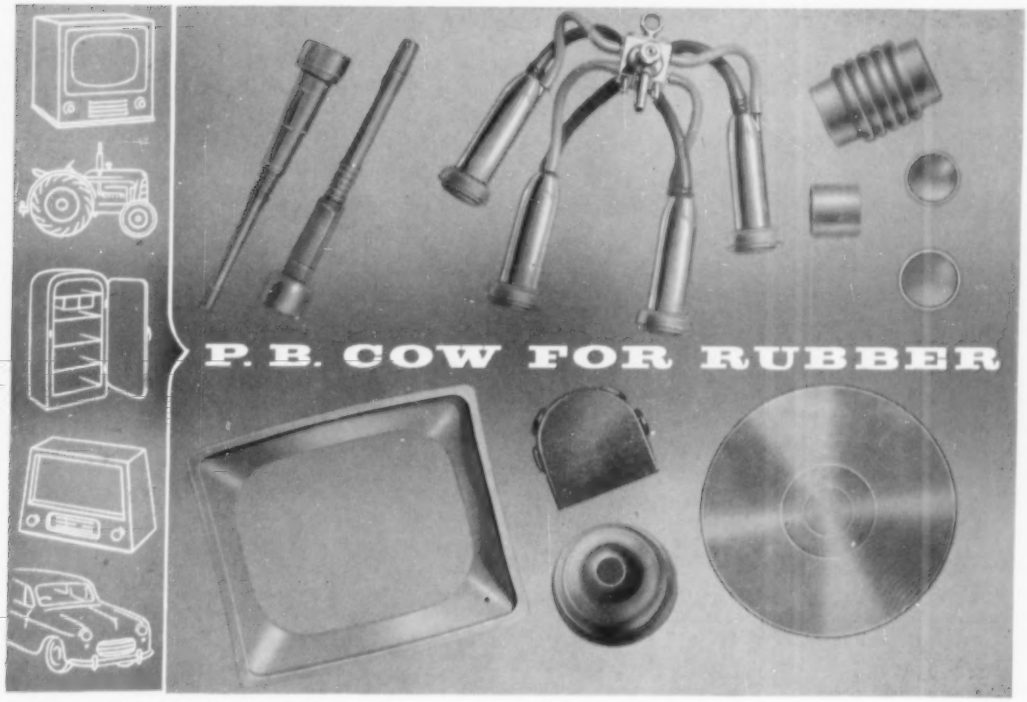

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Excellent opportunities for keen and enthusiastic men in a wide variety of permanent and pensionable positions offering every opportunity for advancement. Salary will be in accordance with age, experience and qualifications.

1. ENGINEERS for development and design of RADIO FREQUENCY GENERATORS. Applicants should preferably have had a full-time electrical apprenticeship and experience in high powered R.F. equipment. Qualifications: Higher National Certificate or full-time university degree.

2. ENGINEERS for development and design of MECHANICAL HANDLING EQUIPMENT associated with RADIO AND MACHINE FREQUENCY HEATING APPLICATIONS. Applicants should preferably have had a full-time electrical or mechanical apprenticeship and have experience in the design of mechanical handling equipment, including pneumatics and hydraulics. Qualifications: Higher National Certificate or full-time university degree.

3. ENGINEERS for development and design of INDUCTION HEATING equipment including vacuum and low pressure furnaces. Applicants should preferably have had a full-time electrical or mechanical apprenticeship. Qualifications: Higher National Certificate or full-time university degree.

4. TECHNICAL ASSISTANTS AND JUNIOR ENGINEERS for Experimental Laboratory work on problems related to the application of RADIO, MACHINE FREQUENCIES AND RESISTANCE HEATING. Candidates should preferably have had a full-time electrical or mechanical apprenticeship and reached at least Ordinary National Certificate standard.

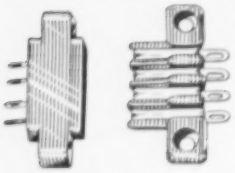
5. DRAUGHTSMEN, Seniors and Juniors, for circuitry layouts in connection with mechanical and electrical equipment associated with RADIO AND MACHINE FREQUENCY AND RESISTANCE HEATING APPLICATIONS. Candidates should preferably have had a full-time electrical or mechanical apprenticeship and hold the Ordinary National or Higher National Certificate in electrical or mechanical engineering.

Write for application form, quoting reference "V.4," to:—

Personnel Manager,
Metropolitan-Vickers Electrical Co., Ltd.,
Trafford Park, Manchester 17

McMURDO

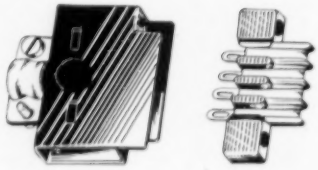
STRIP CONNECTORS



A new line in connectors.
2, 4, 6, 8 and 10 way all in a row.
Ideal for use with printed circuits, the plug can be mounted on the edge of the panel and the contact pin tails bent down for dip soldering.

- Current rating — 5 Amps per contact
- Voltage rating — 750 Volts working

In all but the two-way version, plugs are available for both horizontal and vertical mounting.
Covers with cable clamp can be fitted if required to both plugs and sockets.



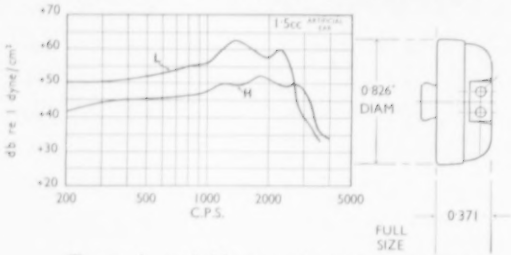
Full details on request

THE McMURDO INSTRUMENT CO. LTD. ASHTEAD, SURREY

Telephone ASHTEAD 3401

SC.1

FORTIPHONE MINIATURE EARPHONES



Types L and H are miniature magnetic earphones of optimum impedance to match small junction transistors such as the OC. 71, XFT. 2, V10/30, EW. 59, or JT. 1.

With a 3 volt supply to the transistor at 8 mA approx. 10 milliwatt input to the earphone will produce the "threshold of pain", if a fitted earmould is used.

impedance at 1000 cps	250 ohms.
d.c. resistance	55 ohms.
sensitivity (L)	170 mv for pressure of + 40 db re 1 dyne/cm².

FORTIPHONE LTD.—COMPONENT DIVISION
247, Regent Street, London, W.1

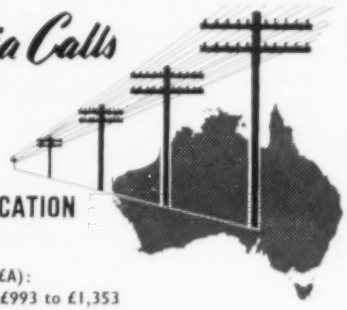
SENIOR ELECTRONIC ENGINEER

Bruce Peebles & Co. Ltd., East Pilton, Edinburgh 5, are forming an Electronic Development Group, and have a vacancy for a Senior Electronic Engineer to take charge of the Group. Applicants should have considerable experience in research and development in electronics and light current engineering. Some service in H.M. Forces or at a research establishment on radar will be advantageous. Qualifications should include an Honours Degree in Electrical Engineering or Physics and preferably corporate membership of the Institute of Electrical Engineers. In general applicants should be in their middle thirties, though a younger man with outstanding capabilities will be considered. The position requires a high degree of responsibility and carries a salary commensurate with this. Applications with full details to Mr. O. L. Robson, Manager, Rectifier Department, at above address.

Senior Telecommunication Engineer required for development of submarine repeaters. Applicants should have several years' experience in carrier telephony practice, e.g., in the design of filters and feedback amplifiers. Technical education: Honours degree in Electrical Engineering or equivalent qualifications.

Post is eligible for Pension Scheme—five-day week. Applicants should give details of age, previous experience and salary required to Personnel Manager, Telcon Works, Greenwich, London, S.E.10.

Australia Calls



TELECOMMUNICATION ENGINEERS

SALARY SCALES (in £A):
Grade I Engineer—£993 to £1,353
Group Engineer—£1,443 to £1,623

AUSTRALIA is developing rapidly and urgently needs electrical, electronic, mechanical and other professionally qualified engineers to assist in the expansion and maintenance of its national telephone, telegraph, radio and television services.

QUALIFICATIONS: Graduate or Corporate Membership of main Institutions of Engineers, or equivalent academic qualifications.

STARTING SALARIES: According to qualifications and experience. University Graduates £993 to £1,128 minimum.

CONDITIONS OF APPOINTMENT: Permanent appointment to the Postmaster-General's Department. Free passages for appointee and dependants. Initial accommodation provided. In approved circumstances, special settling-in allowances. Full salary from date of embarkation. Superannuation, annual and sick leave conditions. 4½ months' furlough after 15 years. Good opportunity for advancement.

FOR FURTHER DETAILS: Engineers qualified or about to qualify, inquire from Australian Post Office Representative, Australia House, Strand, London, W.C.2.

THE GENERAL ELECTRIC COMPANY LTD.,
BROWN'S LANE, ALLESLEY, COVENTRY

require

DEVELOPMENT ENGINEERS
TECHNICAL ASSISTANTS
RADIO or RADAR MECHANICS

for

interesting work on Automation applied to Complex Radar and Guided Missile Systems.

Development Engineers should possess design or development experience in (a) Switching circuits, (b) Data transmission and Servo systems or (c) Radar circuits. A degree in Electrical Engineering, Physics, H.N.C., or equivalent qualifications is desirable.

Technical Assistants should preferably have experience as above. O.N.C. or equivalent is desirable.

Radio or Radar Mechanics for work on prototype equipment. Ability to work from circuit diagrams is essential.

Apply by letter, stating age and experience, to the Personnel Manager. Ref. R.G.

OPPORTUNITIES IN THE UNITED STATES FOR ENGLISH-TRAINED ELECTRONICS ENGINEERS

The Engineering Department, Electronics Division of Westinghouse Electric Corporation, has available a number of attractive situations. Applicants must have professional standing in the Institute of Electrical Engineers or equivalent standing in the Institute of Radio Engineers.

Applicants for these positions should be over 25 years of age with at least three years' experience in design or development in one or more of the following fields:

Circuitry
Ground Radar
Communications
Computers
Filters
Antennas
Microwave Cavities and Plumbing
Missile Guidance Equipment
Navigation Aids

Plant is located in the outskirts of Baltimore, Maryland, in the beautiful Chesapeake Bay area, with splendid facilities for fishing and sailing. The mountains are but an hour and a half's drive away. Modern centrally-heated flats in suburban surroundings are readily available.

Many opportunities for advanced education and for cultural pursuits are available in the Baltimore area.

Generous relocation allowance is provided to help defray engineers' and their dependants' travel and shipping expenses.

To arrange for an interview, send résumé of education, experience and your telephone number to Department W, Westinghouse Electric International Company, 1-3 Regent Street, London, S.W.1. All replies will be treated as confidential.

COLLEGE OF TECHNOLOGY, BIRMINGHAM

DEPARTMENT OF PHYSICS

Applications are invited for the following posts:—

Senior Lecturer

Applicants must have an honours degree and should have some teaching experience. Research or industrial experience is essential. The applicant appointed will be expected to carry out research in pure or applied physics or in electronics and should be capable of supervising the work of research assistants.

Lecturer

Applicants must have good qualifications in physics and should have had some experience of teaching, industry or research. The undertaking of research will be encouraged.

Salaries will be in accordance with the Burnham (Further Education) Scales as follows:—

Senior Lecturers
Men: £1,065 × £25—£1,215.
Women: £852 × £20—£972 (plus equal pay increment).

Lecturers
Men: £965 × £25—£1,065.
Women: £772 × £20—£852 (plus equal pay increment).

Further particulars and forms of application may be obtained from the Registrar, College of Technology, Suffolk Street, Birmingham, 1, on receipt of a stamped addressed foolscap envelope. Completed forms should be returned not later than two weeks after the appearance of this advertisement.

K. R. PILLING,
Clerk to the Governing Body.

B T-H

OPPORTUNITIES FOR ELECTRONICS ENGINEERS

Because of expanding commitments the Company wishes to increase its development engineering staff engaged on radar and other fields of activity. Applications are invited for the positions enumerated below.

1. Microwave systems engineer for interesting work concerning development of the latest techniques in waveguide systems of unusual complexity.
2. Microwave components engineer for the development of components used in complex microwave systems, including novel aerials and feeds.
3. Radar circuits engineers for the development of data extraction systems, including range and velocity processing circuits.
4. Radar transmitter engineer for the development of high power pulse and CW transmitters.
5. I.F. systems engineer for advanced development associated with pulse and Doppler radar.
6. Servo engineers for work concerning:
   (*a*) Developments in automatic machine-tool control.
   (*b*) Atomic energy projects.
7. Mechanical engineer for development work associated with large precision mechanical structures for ground radar systems.

The Company has a wide background of experience in the lines of activity mentioned and an increase in staff is required for large-scale, long-term projects. The posts are permanent and pensionable and offer ample opportunities for interesting work and professional advancement.

Qualifications: Degree in electrical engineering, physics or mechanical engineering as appropriate, and preferably at least two years' practical experience in a related field.

Applications stating age and giving details of experience and qualifications to:

The Manager, Electronics Engineering Dept.,

THE

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LIVE AND WORK

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COUNTRY or by the SEA . . .

SENIOR and JUNIOR DRAUGHTSMEN

required for

Radar, Electronic and Television Projects

The Radar and Communications Division of E. K. COLE LTD. is situated at Malmesbury, in the heart of the Cotswold country. The Television, Radio and Nucleonic Division is at Prittlewell, near Southend-on-Sea.

At Malmesbury assistance is given with housing and travel; at Southend a new drawing office will provide the finest of working conditions.

Interesting work and a pension scheme after a qualifying period is offered to the right man.

Technical education to ordinary certificate standard required, but junior draughtsmen attending O.N.C. classes will be favourably considered.

Write to Chief Draughtsman, E. K. Cole Ltd., Ekco Works, at either Malmesbury, Wilts, or Prittlewell, near Southend-on-Sea.

14th—17th May 1956

A special Appointments Bureau will be open at our London Office, 5 Vigo Street, W.1 (Regent 7030). Write or phone for appointment, or call, between 10 o'clock and 7 o'clock.

Qualified Engineer required to take charge of section concerned with research and development of Tape Recorders. Pensionable appointment. Write with full details of experience and salary required to Box No. A45/6, Strand House, Portugal Street, London, W.C.2.

Design and development engineers required, experienced in radio and television, for work in Jersey, Channel Islands. Salary £900–£1,200 p.a. according to qualifications and experience. Apply Box No. 0938, c/o *Wireless Engineer*.

Physicists and **Electrical Engineers** required by **Ministry of Supply, Radar Research Establishment,** Malvern, Worcs, for research and development, mainly on radar and electronic equipment. Ample scope for initiative and originality over very wide field. Candidates should have 1st or 2nd class hons. degree or equiv. in Physics or Electrical Engineering. Appointments according to age, experience as Senior Scientific Officer (min. age 26 with at least three years' post-graduate research experience) or Scientific Officer (min. age 21). Salary within ranges S.S.O. £1,030–£1,185; S.O. £488 10s.–£885 (Superannuable). Application forms from M.L.N.S., Technical and Scientific Register (K), 26 King Street, London, S.W.1, quoting A148/6A.

COMMERCIAL T.V.

Commercial television and F.M. broadcasting have resulted in vacancies becoming available for men interested in the development of V.H.F. tuners involving new techniques of design and manufacture. Salaries in the range of £650–£1,200 are offered to engineers with the required experience, and prospects of future advancements are good. Write, in confidence, giving full particulars of experience and qualifications to Box No. 1190, c/o *Wireless Engineer*.

TRANSISTOR AND PRINTED CIRCUIT COMPONENTS

Radio Components

The rapid development of transistors and printed circuit techniques has given rise to an interesting field for design and application of new components for these fields. Permanent, pensionable posts are offered to development engineers wishing to undertake advanced development work of this kind. The Company's laboratories, situated in the London area, are fully equipped and offer agreeable working conditions. Write to Box No. 1191, c/o *Wireless Engineer*, details of age, experience and qualifications, which will be regarded as strictly confidential.

Senior Engineer required for **Broadcasting Service, Cyprus,** on contract for two years in first instance. Salary in scale (including overseas allowance and present temporary allowance of 9 per cent of salary) £1,161 rising to £1,548 a year. Gratuity at rate of £100/£150 a year. Free passages. Liberal leave on full salary. Candidates, preferably between 25 and 35, should have sound theoretical knowledge and practical experience of operation and maintenance of MF broadcast transmitters and aerial systems and be able to control and supervise junior staff. Knowledge of studio engineering and recording desirable.

Write to the Crown Agents, 4 Millbank, London, S.W.1. State age, name in block letters, full qualifications and experience and quote M2C/41601/WJ.

UNIVERSITY OF NOTTINGHAM

A Senior Research Studentship, value £500 per annum for not more than three years, is offered as from 1st October 1956, for research in one of the following subjects: Civil, Mechanical, Electrical or Mining Engineering or Metallurgy. Candidates must be graduates and must have not less than two years' industrial experience, or a professional qualification.

Further information and forms of application may be obtained from the Registrar, to whom the completed forms must be returned not later than 1st July 1956.

GOVERNMENT COMMUNICATIONS HEADQUARTERS (FOREIGN OFFICE), CHELTENHAM

Vacancies exist for Mechanical and Electrical Engineers in advanced projects in electronics, wave propagation and communication engineering. Excellent working conditions, backed by well-organized library and technical information services, specialized training and departmental assistance for higher study, considerable freedom in choice of work, unusual opportunities to gain experience of new materials, components and techniques.

Entry status dependent upon age, experience, qualifications, ability; selection by interview; good promotion prospects.

SALARIES—for men in Cheltenham:

Basic Grade Engineer—starting pay according to age up to 34. At 21, £473 10s.; at 34, £930; then by three increments to £1,055.

Main Grade Engineer—£1,055 × £40—£1,335.

Prospects of promotion to Senior Grade (£1,390 × £50—£1,571).

QUALIFICATIONS. Candidates must:

(*a*) hold university degree in physics or electrical or mechanical engineering and have at least two years' practical experience, or

(*b*) be graduates of the Institution of Electrical or Mechanical Engineers, or hold equivalent diploma.

Degree or graduateship examination should embrace telecommunications subjects including radio.

Candidates should have good communications background with knowledge of one or more following subjects:—

LF/HF, VHF and microwave techniques including transmitters, receivers and aerial distribution systems, valves, magnetic tape recorders, printing telegraph systems including VF and time division multiplex; voice communications techniques.

Two or three years' experience in electronics or telecommunications desirable, but not essential.

Appointments unestablished in the first instance but possibility of permanent established appointment by Civil Service Commission selection at a later stage. Apply to Personnel Officer (B17R), 53 Clarence Street, Cheltenham, Glos.

Electrical Engineer or **Physicist** required by **Air Ministry Experimental Establishment** in **Norfolk** for development of airborne radio receiving and transmitting equipments and their associated aerial systems operating mainly on centimetric wavelengths. Applicants should have a 1st or 2nd class hons. degree or equiv. in Electrical Engineering or Physics. Good physics and electronics background desirable. Appointment according to age, experience, etc., as Senior Scientific Officer (min. age 26 with at least three years' post-graduate research experience) or S.O. (min. age 21). Salary within ranges, S.S.O. £1,030-£1,185; S.O. £488 10s.-£885 (Superannuable). Application forms from M.L.N.S., Technical and Scientific Register (K), 26 King Street, London, S.W.1, quoting A134/6A. Closing date 21st May 1956.

Electronic Engineers are required by the Equipment Engineering Department of Westinghouse Brake & Signal Co. Ltd., at Chippenham, Wiltshire, to take part in the development and subsequent engineering design for production, of new control systems, employing electronic techniques, for traffic control on British & Overseas Railways. The scope of the work will range from practical experimental work to the issue of design instructions on which will be based manufacturing drawings, together with the compilation of manufacturing specifications and descriptive pamphlets. In addition, co-operation will be required with Production Engineers during manufacture of first-off equipments in the Company's Chippenham Works.

Positions are available for University graduates with a good telecommunication Degree in Electrical Engineering. Some experience with transistor techniques is required, together with experience of line communication systems employing audio and carrier frequencies. A knowledge of computing techniques would also be an advantage but is not essential.

The salaries will be determined by age, qualifications and experience; the Company operates a pension scheme and a 5-day week.

For further information write giving age and full particulars to Personnel Superintendent, Westinghouse Brake & Signal Co. Ltd., Chippenham, Wilts, quoting ref. EQUIP/N/Q.

UNIVERSITY OF BIRMINGHAM

Department of Electrical Engineering

and

Department of Extra-Mural Studies

Summer School on
Communication Theory, Modulation and Noise

The School is intended for engineers in the electrical communications and allied fields who have taken their degrees without much theoretical work in communications or feel the need for a "refresher" course.

1st July—13th July 1956

At the Department of Electrical Engineering. Tuition £8. Residential accommodation available. Further information: Director of Extra-Mural Studies, The University, Edmund Street, Birmingham, 3.

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ELECTRONICS **BOOKS**
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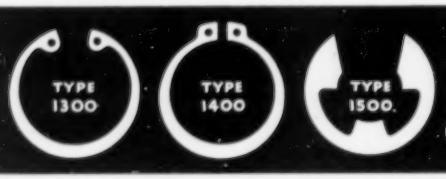
EXPERIENCED RADIO TESTERS AND INSPECTORS

Required for production of communication and radio apparatus. Also instrument makers, wirers and assemblers for factory test apparatus. Apply Personnel Manager, E. K. Cole, Ltd., Ekco Works, Malmesbury, Wilts.

Anderton

Cyril Circlip's stock ranges from 1/16" to 10"



Write for DATA SHEETS to Dept. A.3, ANDERTON SPRINGS LTD., BINGLEY
Telephone: Bingley 2388, 2351 & 2226
Telegrams: Circlips, Bingley

CIRCLIPS

Index to Advertisers

	PAGE
Aero Research, Ltd.	12
Airmec, Ltd.	20
Anderton Springs, Ltd.	28
Appointments Vacant	24, 25, 26, 27, 28
Automatic Coil Winder & Electrical Equipment Co., Ltd., The	8, Cover ii
Belling & Lee, Ltd.	6
British Insulated Callender's Cables, Ltd.	10
Brookes Crystals, Ltd.	22
Cinema Television, Ltd.	2
Cow, P. B. & Co., Ltd.	23
Edison Swan Electric Co., Ltd.	19
Electro-Methods, Ltd.	9
Ferranti, Ltd.	15
Fortiphone, Ltd.	24
Foyle, W. & G., Ltd.	28
General Electric Co., Ltd., The	4
Lewis, H. K. & Co., Ltd.	28
Light Soldering Developments, Ltd.	20
Lyons, Claude, Ltd.	1
McMurdo Instrument Co., Ltd.	24
Metropolitan-Vickers Electrical Co., Ltd.	23
Mullard, Ltd.	16, 18
Multicore Solders, Ltd.	Cover iv
Oxley Developments Co., Ltd.	22
Pitman, Sir Isaac & Sons, Ltd.	20
Plessey, Ltd.	13
Rola-Celestion, Ltd.	3
Salford Electrical Instruments, Ltd.	7
Standard Telephones & Cables, Ltd.	21
Steatite & Porcelain Products, Ltd.	5
Telegraph Condenser Co., Ltd., The	Cover iii
Telephone Mfg. Co., Ltd.	22
Truvox, Ltd.	8
Unbrako Socket Screw Co., Ltd.	11
Welwyn Laboratories, Ltd.	14

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for the Service Engineer

The service engineer depends upon the reliability of the components he uses to safeguard his own reputation: he must be certain they give trouble-free service. The fact that so many service men use only T.C.C. Condensers is evidence of their supremacy. T.C.C. "Lectropack" Dry Electrolytic Condensers are robust yet compact and employ All-Aluminium non-corrosive internal construction. The range shown is a useful guide to the types available, which in the majority of cases will be found the exact replacement.

Capacity μF.	D.C. Volts Wkg.	D.C. Volts Surge	Ripple Current Max. M/A	Dimensions in inches L	Dimensions in inches D	T.C.C. Type No.	List Price
60 - 100	275	325	450	4½	1⅞	CE 37 HE	16/-
60 - 250	,,	,,	530	4½	1⅞	CE 60 HE	28/-
100 - 200	,,	,,	650	4½	1⅞	CE 60 HEA	28/-
100	350	400	450	2¾	1⅜	CE 10 LE	13/6
200	,,	,,	770	4½	1½	CE 36 LE	24/-
60 - 100	,,	,,	450	4½	1⅜	CE 37 LE	21/-
60 - 100	,,	,,	500	4½	1½	CE 36 LEB	23/-
64 - 120	,,	,,	530	4½	1⅞	CE 60 LE	24/-
60 - 250	,,	,,	500	4½	1⅞	CE 60 LEB	34/-
100 - 100	,,	,,	550	4½	1½	CE 36 LEA	26/-
100 - 200	,,	,,	700	4½	1⅞	CE 60 LEA	33/-
100 - 200	,,	,,	700	4½	2	CE 15 LE	34/-
60	450	550	450	3¼	1⅞	CE 38 PE	14/-
60 - 100	,,	,,	500	4½	1⅞	CE 60 PE	29/-

T.C.C. TECHNICAL BULLETIN No. 28 details full range of TV Condensers

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RADIO DIVISION NORTH ACTON LONDON W.3 *Tel:* ACORN 0061

GET 10 TIMES LONGER LIFE OUT OF EVERY SOLDERING IRON BIT USED IN YOUR FACTORY

—CHANGE TO ERSIN MULTICORE Savbit TYPE 1 ALLOY

The bit on the left has been used for making 10,000 soldered joints with Ersin Multicore SAVBIT type 1 alloy. The bit in the centre was used to make 1,000 joints with standard tin/lead alloy. The bit on the right, at the end of its useful life, has made 7,500 joints with standard tin/lead alloy.

Extensive research at the Multicore Laboratories has revealed that the main cause of bit wear is the absorption of copper into the solder alloy. Savbit Alloy which contains a small percentage of copper, prevents this absorption and increases the life of solder bits by about 10 times.

Practical tests on a TV assembly line have shown that these Laboratory investigations are confirmed under actual production conditions.

Ersin Multicore Savbit type 1 Alloy containing 5 Cores of extra active non-corrosive 366 flux is at present available only to manufacturers. It is supplied in 14, 16 and 18 s.w.g. on 7 lb. and 1 lb. reels and costs slightly less than standard 60/40 alloy in a similar gauge.

7 lb. REELS

Available with Ersin Multicore 5 core Solder in 9 gauges and 6 alloys. Prices on application. Finer gauges supplied on 1 lb. and ½ lb. reels. Precision-made Solid Solder Wire also available.

BIB SOLDER THERMOMETER

Designed to indicate temperatures of solder in solder baths and on bits, this simple pyrometer measures temperatures up to 400° C and is calibrated also in degrees Fahrenheit.

BIB RECORDING TAPE SPLICER

For professional and amateur recording tape enthusiasts, this instrument enables tape to be cut and jointed easily, quickly and to extremely fine limits. 18/6 each (subject).

PRINTED CIRCUITS

A complete soldering process has been developed by Multicore Laboratories for the efficient soldering of printed circuits. All details are contained in publication PCL 101. This publication also gives information on a special protective coating formulated to treat tags of components to be solder dipped.

PUBLICATIONS

A revised edition of "Modern Solders" containing valuable information on melting points, standard gauges, constitution of alloys and the use of solder will be gladly sent to laboratory engineers and technicians applying on their firm's official notepaper.

Bib WIRE STRIPPER AND CUTTER

Strips insulation; cuts wire; splits plastic twin flex. A most useful tool for professional and amateur electricians. 3/6 (subject).

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